A COMPENDIOUS SYSTEM OF Natural Philosophy.

With NOTES

Containing the

MATHEMATICAL DEMONSTRATIONS,

AND

Some Occasional REMARKS.

PART I.

The PROPERTIES of BODIES.
Their LAWS of MOTION. And
The MECHANICAL POWERS.

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A COMPENDIOUS SYSTEM OF Natural Philoſophy.

PART I.

The INTRODUCTION.

SO wild and extravagant have been the Notions of a great Part of Philoſophers, both Ancient and Modern, that it is hard to determine, whether they have been more diſtant in their Sentiments from Truth, or from one another; or have not exceeded the Fancies of the moſt fabulous Writers, even Poets and Mythologiſts. This was owing to a precipitate Proceeding in their ſearching into Nature, their neglecting the Uſe of Geometry and Experiment, the moſt neceſſary Helps to the finding out Cauſes, and proportioning them to their Effects.

THE Manner of Philoſophizing among the Ancients was to aſcribe to Bodies certain *arbitrary*

trary Properties, ſuch as beſt ſerv'd their Purpoſe in accounting for the Phænomena * of Nature; from whence proceeded ſo many various Sects of Philoſophers; every one aſſigning a different Cauſe to the ſame Appearance, as his particular Genius and Imagination led him.

The chief Agreement obſervable among moſt of them, conſiſts in this, *viz.* that they conceived all Bodies, as Compoſitions of Air, Earth, Fire, and Water, or ſome one or more of them, from whence they acquired the Name of Principles or Elements, which they ſtill retain.

Epicurus advanc'd a little farther, and aſſerted, that though Bodies conſiſted of ſome one or more of theſe, yet that they were not ſtrictly Elements, but that theſe themſelves conſiſted of Atoms; by an accidental Concourſe of which, (as they were moving through infinite Space in Lines nearly parallel) all Things received their Form and Manner of Exiſtence †.

Des Cartes has contrived an Hypotheſis very different from the reſt; he ſets out with a

* By a Phænomenon of Nature, is meant any Motion or Situation of Bodies among one another, which offers itſelf to the Notice of our Senſes, and is not the immediate Reſult of the Action of an intelligent Being.

† For the Opinions of the Ancient Philoſophers, conſult *Diogenes, Laërtius,* and *Stanley*'s Lives.

Suppo-

Supposition that the Universe at first was entirely full of Matter, that, from this Matter when first put in Motion, there would necessarily be rubbed off (by the grinding of the several Parts one against another) some Particles sufficiently fine to pass through the hardest and most solid Bodies without meeting with any Resistance: of these consists his *Materia subtilis*, or *Materia primi Elementi*. He imagined that from hence also would result other Particles of a globular Form, to which he gave the Name of *Materia secundi Elementi.* Those which did not so far lose their first Figure, as to come under the Denomination of *Materia primi* or *secundi Elementi*, he called *Materia tertii Elementi*; and maintain'd that all the Variety, which appears in natural Bodies, was owing to different Combinations of those Elements.

He likewise supposes that God created a certain Quantity of Motion, and assigned it to this Mass of Matter, and that That Motion (being once created) could no more be annihilated without an omnipotent Hand, than Body itself; in Consequence of which, he was obliged to teach, that the Quantity of Motion is always the same: So that if all the Men and Animals in the World were moving, yet still there would be no more Motion, than when they were at Rest, the Motion which they had not, when at Rest, being

transferr'd to the Æther. So unaccountable are the Notions of this great Philoſopher, that it is ſurprizing his Doctrine ſhould have met with ſuch univerſal Reception, and have got ſo ſtrong a Party of Philoſophers on its Side, that notwithſtanding it was more abſurd, than the Schoolmens *Subſtantial Forms*, they muſt all be exploded to make Way for his ingenious Hypotheſis.

DES CARTES has been ſaid by a late Writer *, to have joined to his great Genius an exquiſite Skill in Mathematicks, and by mixing Geometry and Phyſics together, to have given the World Hopes of great Improvements in the latter. But this Writer ought to have conſidered that what he look'd upon in DES CARTES's Book of Principles, as Demonſtrations, are only Illuſtrations, there not being a Demonſtration from Geometry in all his Philoſophical Works †.

THE preſent Method of Philoſophizing eſtabliſh'd by Sir ISAAC NEWTON, is to find out the Laws of Nature by Experiments and Obſervations. To this, with a proper Application

* Mr. *Wotton* in his Reflections on Ancient and Modern Learning.

† See this Subject diſcuſs'd more at large in *Keil*'s Introduction to his Examination of Dr. *Burnet*'s Theory.

tion

tion of Geometry, is owing the great Advantage the preſent Syſtem of Philoſophy has over all the preceding ones, and the vaſt Improvement it has received within the laſt Age. It is indeed in vain to imagine, that a Syſtem of Natural Philoſophy can be framed by any other Method: for without Obſervations it is impoſſible we ſhould diſcover the Phænomena of Nature, without Experiments we muſt be ignorant of the mutual Actions of Bodies, and without Geometry we can never be certain whether the Cauſes we aſſign be adequate to the Effects we would explain, as the various Syſtems of Philoſophy built on other Foundations, evidently ſhew.

THIS Way of ſearching into Nature was firſt propoſed by my Lord BACON *, proſecuted by the *Royal Society*, the *Royal Academy* at *Paris*, the Honourable Mr. BOYLE, Sir ISAAC NEWTON, &c.

WHAT wonderful Advancement in the Knowledge of Nature may be made by this Method of Enquiry, when conducted by a Genius equal to the Work, will be beſt underſtood by conſidering the Diſcoveries of that excellent Philoſopher laſt mentioned. To Him it is principally owing, that we have now a

* See his *Novum Organum.*

rational

rational Syſtem of Natural Philoſophy; 'tis He who, by purſuing the ſure and unerring Method of reaſoning from Experiment aud Obſervation, joined with the moſt profound Skill in Geometry, has carried his Enquiries to the moſt minute and inviſible Parts of Matter, as well as to the largeſt and moſt remote Bodies in the Univerſe, and has eſtabliſhed a Syſtem not ſubject to the Uncertainty of a mere Hypotheſis, but which ſtands upon the ſecure Baſis of Geometry itſelf.

CHAP.

CHAP. I.

The Properties of Body.

IT being the Deſign of *Phyſics* or Natural Philoſophy to account for the Phænomena of the Material World, it is neceſſary to begin with laying down the known Properties of Body.

THESE are 1. Solidity. 2. Extenſion. 3. Diviſibility. 4. A Capacity of being moved from Place to Place. 5. A Paſſiveneſs or Inactivity. Which are the eſſential Properties of Body; as appears from what follows.

1. SOLIDITY, called alſo Impenetrability, is that Power which Body has of excluding all others out of its Place.

THAT Body, as ſuch, muſt be endued with this Property follows from its Nature, for otherwiſe two Bodies might exiſt in the ſame Place, which is abſurd. The ſofteſt are equally ſolid with the hardeſt, for we find by Experiment, that the Sides of a Bladder, filled with Air or Water, can by no Means be made to come cloſe together *.

* At *Florence* a hollow Globe of Gold was filled with Water, and then exactly cloſ'd; the Globe thus cloſed was put into a Preſs driven by the Force of Screws; the Water, finding no Room for a nearer Approach of its Particles toward each other, made its Way through the Pores of that cloſe Metal,

2. THAT Body is extended, is ſelf-evident; it being impoſſible to conceive any Body, which has not Length, Breadth, and Thickneſs, that is, Extenſion.

3. IT is no leſs evident, that Body is diviſible, for ſince no two Particles of Matter can exiſt in the ſame Place, it follows that they are really diſtinct from each other, which is all that is meant by being diviſible.

IN this Senſe the leaſt conceivable Particle muſt ſtill be diviſible, ſince it will conſiſt of Parts, which will be really diſtinct *. To illuſtrate this by a familiar Inſtance: Let the leaſt imaginable Piece of Matter be conceived lying on a ſmooth plain Surface, 'tis evident the Surface will not touch it every where, thoſe Parts therefore, which it does not touch, may be ſuppoſed ſeparable from the other, and ſo on as far as we pleaſe; and this is all that is

Metal, ſtanding in Drops like Dew on the Outſide, before the Globe would yield to the violent Preſſure of the Engine. V. *Acad. del Ciment.*

* This Propoſition is demonſtrated Geometrically thus, ſuppoſe the Line *A D* (*Fig.* 1.) perpendicular to *B F*, and another as *G H* at a ſmall Diſtance from it alſo perpendicular to the ſame Line; with the Centers *CCC*, &c. deſcribe Circles cutting the Line *G H* in the Points *e, e, e,* &c. Now the greater the Radius *AC* is, the leſs is the Part *e H*. But the Radius may be augmented in infinitum. So long therefore the Part *E H* may be divided into ſtill leſs Portions; conſequently it may be divided in infinitum. *Q. E. D.* V. *Keil*'s Introd. ad Phyſ. Præl. 3, 4, 5, *Graveſande*'s Elem. Math. Phyſ. L. 1. c. 4. Schol.

is meant, when we ſay Matter is infinitely diviſible.

How far Matter may actually be divided, may in ſome manner be conceiv'd from hence* that a Piece of Wire, gilt with ſo ſmall a Quantity as eight Grains of Gold, may be drawn out to a Length of thirteen Thouſand Feet, the whole Surface of it ſtill remaining cover'd with Gold †.

A Quantity of Vitriol, being diſſolved and mix'd with nine Thouſand Times as much Water, will tinge the whole, conſequently the Vitriol will be divided into as many Parts as there are viſible Portions of Matter in that Quantity of Water §.

THERE are Perfumes, which, without a ſenſible Diminution of their Quantity, ſhall fill a very large Space with their odoriferous Particles, which muſt therefore be of an inconceiveable Smallneſs, ſince there will be a ſuffi-

* We have a ſurprizing Inſtance of the Minuteneſs of ſome Parts of Matter, from the Nature of Light and Viſion. Let a Candle be lighted and placed in an open Plane, it will then be viſible two Miles round, conſequently was it placed two Miles above the Surface of the Earth, it would fill with luminous Particles a Sphere, whoſe Diameter was four Miles, and that before it had loſt any ſenſible Part of its Weight. The Force of this Argument will appear better when the Reader is acquainted with the Cauſe of Viſion.

† *Keil*'s Introd. ad Phyſ. Præl. 5. Religious Philoſ. Contempl. 25.

§ Mem. de l'Acad. 1706.

cient Number in every Part of that Space, ſenſibly to affect the Organ of Smelling.

4. THAT all Matter is moveable, follows from its being finite; and to ſuppoſe it poſitively infinite is abſurd, becauſe it conſiſts of Parts *.

5. BY the Paſſiveneſs or Inactivity of Matter, (commonly call'd its *Vis Inertiæ*) is meant the Propenſity it has to continue its State of Motion or Reſt, till ſome external Force acts upon it. This will be farther explained under the firſt Law of Nature.

CHAP. II.

Of Vacuum.

I. PLACE void of Matter is called empty Space, or *Vacuum*.

II. IT has been the Opinion of ſome Philoſophers, particularly the *Carteſians*, that Nature admits not a *Vacuum*, but that the Univerſe is entirely full of Matter: in conſequence of which Opinion they were oblig'd to aſſert, that if every Thing contain'd in a Veſſel could be taken out or annihilated, the Sides of that Veſſel, however ſtrong, would come together; but this is contrary to Expe-

* See Mr. *Law*'s Tranſlation of ABp. *King de Origine Mali.* Not. 3

rience,

rience, for the greateſt Part || of the Air may be drawn out of a Veſſel by means of the Air-Pump, notwithſtanding which it will remain whole, if its Sides are ſtrong enough to ſupport the Weight of the incumbent Atmoſphere.

III. SHOULD it be objected here, that as it is impoſſible to extract all the Air out of a Veſſel, and that there will not be a *Vacuum* on that Account; the Anſwer is, that ſince a very great Part of the Air, that was in the Veſſel, may be drawn out, as appears by the more quick Deſcent of light Bodies in a Receiver * when exhauſted of its Air, there muſt be ſome Vacuities between the Parts of the remaining Air: which is ſufficient to conſtitute a *Vacuum*. Indeed to this it may be objected by a *Carteſian*, that thoſe Vacuities are fill'd with *Materia ſubtilis*, that paſſes freely through the Sides of the Veſſel, and gives no Reſiſtance to the falling Bodies: but as the Exiſtence of this *Materia ſubtilis* can never be prov'd, we are not oblig'd to allow the Objection; eſpecially ſince Sir ISAAC NEWTON has found, that all Matter affords a Reſiſtance nearly in Proportion to its Denſity †.

THERE are many other Arguments to prove this, particularly the Motions of the Comets

|| A Veſſel cannot be entirely exhauſted of its Air, becauſe the Action of the Pump depends on the Spring of that which remains in the Veſſel.

* By this Term is meant any Veſſel, out of which we extract the Air by the Air-Pump.

† *Newt.* Principia Lib. 2. Prop. 31. & 40. & Opt. Edit. 2. Book 3. Quer. 18, 19, 20, 21. *Deſagul.* Lect. 1. Ann. 2.

through the Heavenly Regions without any ſenſible Reſiſtance *; the different Weight of Bodies of the ſame Bulk, &c. but thoſe, being not yet explain'd, are not ſo proper to be inſiſted on in this Place.

Of Attraction and Repulsion.

CHAP. III.

I. BESIDES the forementioned Properties of Matter, it has alſo certain Powers or active Principles, known by the Names of *Attraction* and *Repulsion*, probably not eſſential or neceſſary to its Exiſtence, but impreſſed upon it by the Author of its Being, for the better Performance of the Offices for which it was deſign'd.

II. ATTRACTION is of two Kinds. 1. Coheſion, or that by which minute Bodies, (or the ſeveral Particles of the ſame Body) when placed aſunder at very ſmall Diſtances, mutually approach each other; and then adhere or ſtick together, as if they were but one. 2. Gravitation, or that by which diſtant Bodies act upon each other.

III. THE Attraction of Coheſion is prov'd from abundance of Experiments, of which ſome of the moſt obvious are as follows.

* *Deſagul.* Lect. 1. Annot. 8.

1. LET a small glass Tube (commonly call'd a Capillary Tube) open at both Ends, be dipt into a Vessel of Water, the Water will immediately rise up in the Tube to a certain Height above the Level of the external Water. This Rise of the Water in the Glass Tube is manifestly owing to the Attraction of those Particles of the Glass, which lie in the inner Surface of the Tube immediately above the Water: Accordingly the Quantity of Water raised is always found to be proportionable to the Largeness of that Surface *.

2. LET two Spheres of Quicksilver be placed near each other, and they will immediately run together, and form one Globule.

IV. THE Laws of this Attraction are, 1. That it acts only upon Contact, or at very small Distances; for the Spheres, mentioned in the last Experiment, will not approach each other,

* The Heights the Water rises to in different Tubes, are observed to be reciprocally as the Diameters of the Tubes, from whence it follows that the Quantities raised are as the Surfaces which raise them.

Dem. Let there be two Tubes, the Diameter of the first double to that of the second, the Water will rise half as high in the first as in the second: now was it to rise equally high in both, the Quantity in the first would be four times as great as in the second, (Cylinders of equal Heights being as the Squares of their Diameters; 11. *El.* 14.) therefore since it is found to rise but half as high, the Quantity is but twice as much, and therefore as the Diameter; but the Surfaces of Cylinders are as their Diameters, therefore the Quantities of Water raised are also as the Surfaces. *Q. E. D.*

See the Dissertation on this Subject. Part II.

till

till they are plac'd very near. 2. It acts according to the Breadth of the Surfaces of the attracting Bodies, and not according to their Quantities of Matter. For, let there be two polish'd Glass Plates laid one upon another, in such a Manner as to touch at one End, and there make a very small Angle: If two unequal Drops of Oil be put between these Plates, at equal Distances from the Line of Contact, so that the least may touch both Glasses, they will then both move towards the Ends that touch, because the Attraction of the Surfaces inclines that Way; but the largest, touching the Glasses in most Points, will move the fastest. 3. 'Tis observ'd to decrease much more than as the Squares of the Distances of the attracting Bodies from each other increase: That is, whatever the Force of Attraction is at a given Distance, at twice that Distance it shall be more than four Times less than before *.

V. From hence it is easy to account for the different Degrees of Hardness in Bodies; those whose constituent Particles are flat or square, and so situated as to touch in many Points, will be hard; those Particles which are more round, and touch in fewer Points, will constitute a softer Body; those which are spherical, or nearly of that Figure, will form a Fluid. †.

* V. *Keilii* Opera Ed. 4*to. p.* 626.

† See *Rohault* in the Notes, p. 105, 108. See Part II. Chap. I. §. 2. in the Notes. *Newtoni.* Optic. p. 335.

VI. At-

VI. Attraction of Gravitation is that, by which distant Bodies act upon each other. Of this we have daily Instances in the falling of heavy Bodies towards the Earth.

VII. The Laws of this Attraction are, 1. That it decreases, as the Squares of the Distances between the Centers of the attracting Bodies increase. Thus, a Body, which at the Surface of the Earth (*i. e.* about the Distance of four Thousand Miles from its Center) weighs ten Pounds, if it was plac'd four Thousand Miles above the Surface of the Earth, *i. e.* twice as far distant from the Center as before, would weigh four Times less; if thrice as far, nine Times less, *&c.* The Truth of this Proposition is not to be had from Experiments, (the utmost Distance we can convey Bodies to, from the Surface of the Earth, bearing no Proportion to their Distance from its Center,) but is sufficiently clear from the Motions observ'd by the heavenly Bodies. 2. Bodies attract one another with Forces proportionable to the Quantities of Matter they contain; for all Bodies are observ'd to fall equally fast in the exhausted Receiver, where they meet with no Resistance. From whence it follows, that the Action of the Earth upon Bodies is exactly in Proportion to the Quantities of Matter they contain; for was it to act as strongly upon a less Body as upon a larger, the least Body, being most easily put into Motion, would

would move the fastest. Accordingly, it is observable, that the Weight of a Body is the same whether it be whole, or ground to Powder*.

VIII. From hence it follows, that, was a Body to descend from the Surface toward the Center of the Earth, it would continually become lighter and lighter, the Parts above attracting it, as well as those below; in which Case it is demonstrated by Mathematicians, that the Gravity would decrease with the Distance of the Body from the Center †.

Scholium.

* *Gravesande* Lib. 4. Chap. 11. *Cotes*'s Preface to *Newton*'s Princip.

† *Dem.* Let there be a Body as *P.* (*Fig.* 2.) placed any where within a Concave Sphere, as *AB*, which let us suppose divided into an infinite Number of thin concentric Surfaces; I say, the Body P will be attracted equally each way by any one of these *v. g.* the interior *HIKLM*. Let there be Lines as *IL*, *HK*, &c. drawn through any Point of the Body *P*, in such a manner as to form the Surface of two similar Figures; suppose Cones, the Diameters of whose Bases may be *IH*, *KL*, which let be infinitely small. These Bases (being as the Squares of the Lines *IH*, *KL*) (2. *Elem.* 12.) will be directly, as the Squares of their Distances from *P* (for the Triangles *IPH*, *KPL*, being infinitely small, are similar.) But those Bases include all the Particles of Matter in the interior Surface, that are opposite to each other; the opposite Attractions are therefore in the same Ratio with those Bases, that is as the Squares of the Distances *PK*, *PI*. But the Attraction is inversely, as the Squares of the Distances of the attracting Bodies, §. 7. *i. e.* inversely as the Squares of the same Distances *PK PI*; these two Ratios therefore destroying each other, it is evident, that if the Concavity of the Sphere was filled with Matter, that alone, which lies nearer the Center than the Body can effect it, the respective Actions of all the Parts, that are more distant, being equal, and

Scholium. IT may be proper to obſerve here, that when Philoſophers ſpeak of Bodies gravitating to, or attracting each other, that Body is ſaid to gravitate to another, which moves towards it, while the other actually is, or appears to be, at reſt, and this other is ſaid to attract the former; though indeed the Force being mutual and equal on both Sides (as will be explain'd under the third Law of Nature) the ſame Term might be apply'd to either the gravitating or attracting Body.

IT is farther to be obſerv'd, that when we uſe the Terms, Attraction or Gravitation, we do not thereby determine the Phyſical Cauſe of it, as if it proceeded from ſome ſuppoſed *occult* Quality in Bodies; but only uſe thoſe Terms to ſignify an Effect, the Cauſe of which lies out of the reach of our Philoſophy. Thus, we may ſay, that the Earth attracts heavy Bodies; or that ſuch Bodies tend or gravitate to the Earth: although at the ſame time we

and in contrary Directions, ſince the ſame is demonſtrable of any of the remaining concentric Surfaces. Let us ſee then what Effect that, which lies nearer the Center than the Body, will have upon it, which may be conſidered as a Sphere, on whoſe Surface the Body is plac'd. The Diſtances of each Particle of Matter from the Body, (taken collectively) will be as the Diameter of the Sphere, or as the Radius, *i. e.* as the Diſtance of the Body from the Center: their Action therefore upon the Body will be inverſely as the Square of that Diſtance: but the Quantity of Matter will be as the Cube of that Diſtance, (18. *Elem.* 12.) the Attraction therefore will be alſo in that Proportion. Now, theſe two Ratios being compounded, the Attraction will be only as the Diſtance of the Body from the Center. *Q. E. D.*

are wholly ignorant: whether this is effected by ſome Power, actually exiſting in the Earth, or in the Bodies, or external to both; ſince it is impoſſible any Error in our Reaſonings can follow from hence: it being evident, that all the Conſequences of ſuch Tendency muſt be the ſame, let the Cauſe be where, or what it will.

X. Repulsion is that Property in Bodies, whereby, if they are placed juſt beyond the Sphere of each other's Attraction of Coheſion, they mutually fly from each other.

Thus, if an oily Subſtance, lighter than Water, be placed on the Surface thereof, or if a Piece of Iron be laid on Mercury, the Surface of the Fluid will be depreſs'd about the Body laid on it: This Depreſſion is manifeſtly occaſion'd by a repelling Power in the Bodies, which hinders the Approach of the Fluid towards them.

But it is poſſible in ſome Caſes to preſs or force the repelling Bodies into the Sphere of one another's Attraction; and then they will mutually tend towards each other; as when we mix Oyl and Water till they incorporate *.

XI. Besides the general Powers forementioned, there are ſome Bodies that are endued with another, call'd *Electricity*. Thus Amber,

* We have an undeniable Proof of this Repulſive Force in Sir *Iſaac Newton*'s Opticks. *B*. 3. and Query 31.

Jet,

Jet, Sealing-Wax, Agate, Glaſs, and moſt Kinds of precious Stones attract and repel light Bodies at conſiderable Diſtances.

THE chief Things obſervable in theſe Bodies are, 1. That they don't act, but when heated. 2. That they act more forcibly when heated by rubbing, than by Fire. 3. That, when they are well heated by rubbing, light Bodies will be alternately attracted and repell'd by them, but without any obſervable Regularity whatever. 4. If a Line of ſeveral Yards in length has a Ball, or other Body ſuſpended at one End, and the other End be fixed to a Glaſs Tube; when the Tube is heated by rubbing, the Electrical Virtue of the Glaſs will be communicated from the Tube to the Ball, which will attract and repel light Bodies in the ſame Manner, as the Glaſs itſelf does. 5. If the Glaſs Tube be emptied of Air, it loſes its Electricity *.

XII. LASTLY, the Loadſtone is obſerv'd to have Properties peculiar to itſelf, as that by which it attracts and repels Iron, the Power it communicates to the Needle, and ſeveral others †.

* See *Haukſbee's* Experiments. *Philoſoph. Tranſact.* No. 326.

† Several Solutions of theſe Properties of *Electricity* and *Magnetiſm* have been attempted by different Philoſophers, but all of them ſo unſatisfactory as not to deſerve a particular Account in this Place. See *Chambers's* Dictionary in *Electricity*, and *Des Cartes* Opera Philoſophica. P. IV. §. 133, with ſeveral others referr'd to in *Quæſtiones Philoſophicæ. Deſagul.* Lect. I. §. 33.

CHAP. IV.

Of the Laws of Motion, commonly called Sir ISAAC NEWTON'S *Laws of Nature.*

I. ALL Bodies continue their State of Reſt, or uniform Motion in a right Line, till they are made to change that State by ſome external Force impreſſed upon them.

THIS Law is no other, than that univerſal Property of Bodies, called Paſſiveneſs or Inactivity; whereby they endeavour to continue the State they are in, whatever it be. Thus a Top only ceaſes to run round on Account of the Reſiſtance it meets with from the Air, and the Friction of the Plane whereon it moves. And a Pendulum, when left to vibrate in *vacuo*, where there is nothing to ſtop it, but the Friction ariſing from the Motion of the Pin on which it is ſuſpended, continues to move much longer, than one in the open Air.

II. THE Change of Motion produc'd in any Body, is always proportionable to the Force, whereby it is effected; and in the ſame Direction, wherein that Force acts.

THIS is an immediate Conſequence of this Axiom, the Effect is always proportionable to its

its Cause. For Instance, if a certain Force produces a certain Motion, a double Force will produce double the Motion; a triple Force triple the Motion, &c. If a Body is in Motion, and has a new Force impressed on it in the Direction wherein it moves, it will receive an Addition to its Motion, proportional to the Force impressed; but if the Force acts directly contrary to its Motion, the Body will then lose a proportional Part of its Motion: Again, if the Force is impressed obliquely, it will produce a new Direction in the Motion of the Body, more or less different from the former in Proportion to its Quantity and Direction *.

III. RE-

* This Case is expressed more accurately by Mathematicians thus. If the Proportion and Direction of two Forces, acting upon a Body at the same Time, be represented by the Sides of a Parallelogram, the Diagonal of that Parallelogram will represent the Proportion and Direction of their united Forces.

Dem. Let the Body *A* (*Fig.* 3.) be impell'd with a Force, which would carry it to *E*, in the same Time that another, acting upon it in the Direction *AD*, would carry it to *D*. Imagine that while the Body passes to *E*, the Line *AD* (in which the Body moves by the other Force) moves to *EB*, in a Direction parallel to itself; when the Body has advanc'd to *G* in the Line *AE*, the Line *AD* will have got to *GF*, and the Body will have passed over *GH*, such a Part of it, as bears the same Proportion to the whole Line GF, as *AG* does to *AE*, that is, *GH* (the shorter Side of the Parallelogram *GM*,) is to *GF*, or, which is the same Thing, to *EB* (the shorter Side of the Parallelogram *ED*) as *AG* (the longer Side of the former) is to *AE* the longer Side of the latter,) from whence the Parallelograms are similar, *El.* 6. *Def.* 1. and consequently, by 24 *El.* 6. the Point *H* is in the Diagonal, that is, the Body will always be found in the Line *AB*. Q.E.D.

Coroll.

III. Reaction is always contrary, and equal to Action; or the Actions of two Bodies upon each other, are equal, and in contrary Directions.

Thus, suppose a Stone, or other Load to be drawn by an Horse; the Load reacts upon the Horse, as much as the Horse acts upon the Load; for the Harness, which is stretched equally between them both Ways, draws the Horse towards the Stone, as much as it does the Stone towards the Horse; and the progressive Motion of the Horse is as much retarded by the Load, as the Motion of the Load is promoted by the Endeavour of the Horse*. This will be better explained from the following Instance; let a Person, sitting in a Boat,

Coroll. From hence we have an easy Method of resolving a given Motion into any two, or more Directions whatever; *viz.* by describing a Parallelogram about the given Direction as a Diagonal, the two Sides of which will represent the Directions sought. Thus, suppose a Body was impell'd in the Line *AB*, we may conceive it as acted upon by two Forces at the same Time, one towards *E*, the other towards *D*, or any other two whatever, provided the Lines be drawn of such length, that, when the Parallelogram is compleated, the given Line *AB* shall be its Diagonal.

* It may be thought perhaps, that (two equal and contrary Forces destroying one another) the Horse will in this Case not be able to move at all, because the Load draws him back, as much as he draws the Load forwards. But it is to be obved that the Strength of the Horse is not properly exerted upon the Load but upon the Ground; and consequently the Ground, reacting and continuing at Rest, pushes the Horse forward with just so much Force as the Horse exerts, above what is counteracted by the Load.

draw

draw another Boat equally heavy towards him, they will both move towards each other with equal Velocities: Let the Boat he ſits in be the lighteſt, and it will move the faſteſt; becauſe the Action being equal on both Sides, the ſame Quantity of Motion will be given to each Boat, that is, the leſſer will have the greater Velocity *.

WE have a farther Confirmation of this from Attraction. Suppoſe two Bodies attracting one another, but prevented from coming cloſe together by ſome other Body placed between them: If their reſpective Actions, by which they tend towards each other, were not equal on both Sides, then would the intermediate Body be preſſed more one Way than the other, and ſo all three would begin to move of themſelves the ſame Way; but that three Bodies ſhould be put into Motion after this Manner, when no external Force acts upon them, is contrary to Experience, conſequently whatever different Degres of Force, any two Bodies may be capable of exerting, their mutual Actions on each other, are always equal. This may be try'd with a Loadſtone and Iron; which, being put into proper Veſſels, contiguous to one another, and made to float on the Surface of Water, will be an exact Counterbalance to

* See the Diſtinction between Motion and Velocity. Chap. 9.

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each

each other, and remain at Reſt, whatever be the attractive Power of the Loadſtone, or the Proportion of their reſpective Magnitudes.

THESE Laws receive an abundant Additional Proof from hence, *viz.* that all the Concluſions that are drawn from them, in Relation to the Phænomena of Bodies, how complicated ſoever their Motions be, are always found to agree perfectly with Obſervation. The Truth of which ſufficiently appears in all Parts of the *Newtonian* Philoſophy *.

CHAP. V.

The Phænomena of Falling Bodies.

I. THE Laws of Nature being thus explained, we proceed to account for ſome of thoſe Phænomena, which are ſolvable by them.

II. To begin with thoſe of falling Bodies. Conſtant Experience ſhews, that Bodies have a Tendency towards the Earth, which is call'd Gravity, the Laws of which were enumerated in Chap. 3. §. 7.

III. THE Height, Bodies can be let fall from, bears ſo ſmall a Proportion to their Diſtance from the Center of the Earth, that it cannot

* See theſe Laws explain'd more at large by *Cheyne* in his Principles of Philoſophy. *Keil's* Introd. ad Phyſ. Præl. 11, 12.

ſenſibly

ſenſibly alter their Gravity; which therefore may be conceiv'd, as acting conſtantly and uniformly upon them, during the whole Time of their Fall: From whence they muſt neceſſarily acquire, at every Inſtant, an equal Degree of Velocity, which on that Account will conſtantly increaſe, in Proportion to the Time the Body takes up in falling.

IV. THE Spaces Bodies fall through in different Times, reckoning from the Beginning of their Fall, are as the Squares of thoſe Times; thus, a Body will fall four Times as far in two Minutes, as it does in one, and nine Times as far in three, ſixteen Times as far in four, &c.*

V. FROM

* In order to demonſtrate this Propoſition, it will be neceſſary to lay down the following Theorem, *viz.*

That the Space a Body paſſes over, with an uniform Motion, is in a Ratio compounded of the Time and Velocity. For the longer a Body continues to move uniformly, the more Space it moves over; and the faster it moves during any Interval of Time, the farther it goes; therefore the Space is in a Ratio compound of both, that is, is had by multiplying one into the other.

Coroll. Therefore the Area of a Rectangle, one of whoſe Sides repreſents the Celerity a Body moves with, and the other the Time of its Motion, will expreſs the Space it moves through.

This being premiſed, let the Line *AB* (*Fig.* 4.) repreſent the Time a Body takes up in falling, and let *BC* expreſs the Celerity acquir'd by its Fall; farther let the Line *AB* be divided into an indefinite Number of ſmall Portions, *ei*, *im*, *mp*, &c. and let *ef*, *ik*, *mn*, *pq*, &c. be drawn parallel to the Baſe. Now it is evident from § 3. (*viz.* that the Velocities are as the Times in which they are acquir'd) that the Lines *ef*, *ik*, *mn*, *pq*, &c. being to each other (4. *El.* 6.) as the Lines *Ae*, *Ai*, *Am*, *Ap*, &c. will repreſent the Celerities in the Times repreſented by theſe: that is, *ef* will be as the Velocity of the

Body

V. From this Propoſition it follows, that a Body falls three Times as far, in the ſecond Portion of Time, as it does in the firſt; five Times as far in the third; ſeven Times in the fourth, and ſo on in the Series of the odd Numbers: For otherwiſe it could not fall four Spaces in two Minutes, and nine in three, as the Propoſition aſſerts *.

VI. The

Body in the ſmall Portion of Time *ei*, and *ik* will be as the Velocity in the Portion of Time *im*; in like Manner *pq* will be as the Velocity in the Portion of Time *po*, which Portions of Time being taken infinitely ſmall, the Velocity of the Body may be ſuppos'd the ſame, during any whole Portion: and conſequently, by the Corollary of the foregoing Theorem, the Space run over in the Time *ei* with the Velocity *ef* may be repreſented by the Rectangle *if*: in like Manner the Space run over in the Time *im*, with the Celerity *ik*, may be expreſs'd by the Rectangle *mk*; and that run over with the Celerity *mn* in the Time *mp*, by the Rectangles *pn*; and ſo of the reſt. Therefore the Space run over in all thoſe Times will be repreſented by the Sum of all the Rectangles, that is, by the Triangle *ABC*, for thoſe little triangular Deficiencies, at the End of each Rectangle, would have vaniſhed, had the Lines *ei*, *im*, *mp*, &c. been infinitely ſhort, as the Times they were ſuppoſed to repreſent. Now as the Space, the Body deſcribes in the Time *AB*, is repreſented by the Triangle *ABC*, for the ſame Reaſon the Space paſs'd over in the Time *Ao* may be repreſented by the Triangle *Aor*, but theſe Triangles, being ſimilar, are to each other, as the Squares of their homologous Sides *AB* and *Ao* (20 *El.* 6): that is, the Spaces repreſented by the Triangles are to each other, as the Squares of the Times repreſented by the Sides. *Q. E. D.*

* This may alſo be ſhewn in the following Manner. Let the Triangle *ABC* (*Fig.* 4.) be divided into leſſer ones, as in *Fig.* 5. each equal to *Dbr*, which repreſents the Space deſcribed by the falling Body in *Db* the firſt Portion of Time; 'tis evident that, in *bc* the ſecond Portion of Time, there are three ſuch Triangles deſcribed, *viz.* thoſe that lie between the

Lines

VI. The Spaces, deſcrib'd by falling Bodies in different Times, are as the Squares of the laſt acquir'd Velocities. For by §. 4. the Spaces are as the Squares of the Times, and by §. 3. the Velocities are as the Times; therefore the Spaces are alſo as the Squares of the Velocities.

VII. The Space a Body paſſes over, from the Beginning of its Fall in any determinate Time, is half what it would deſcribe in the ſame Time moving uniformly with its laſt acquir'd Velocity *.

VIII. In like Manner, when Bodies are thrown up perpendicularly, their Velocities decreaſe, as the Times they aſcend in increaſe; their Gravity deſtroying an equal Portion of their Velocity every Inſtant of their Aſcent.

IX. The Heights Bodies riſe to, when thrown perpendicularly upwards, are as the Squares of the Times ſpent from their firſt ſetting out, to the Moment they ceaſe to riſe. That is, if a Body is thrown with ſuch a Degree of Velocity, as to continue riſing twice as

Lines *br* and *cs*; in *cd* the third Portion of Time, five ſuch, *viz.* all between *cs* and *dt*; in *df* the next equal Portion of Time, ſeven ſuch, &c.

* For let the Time be *AB*, (*Fig.* 4.) and the laſt Velocity *BC*, the Space the Body runs over, while it is acquiring that Velocity, is as *ABC*, but the Space it would paſs over in the Time *AB*, was it to move uniformly with the Celerity *BC*, is by the Theorem (Note p. 25.) as the Space *ABCD*, double the former. *Q. E. D.*

long as another, it ſhall aſcend four Times as high; if thrice, nine Times as high, &c.

THESE two are the converſe of the third and fourth Sections *

CHAP. VI.

Of the Deſcent of Bodies on oblique Planes, and of Pendulums.

WHEN a Body deſcends on an oblique Plane, its Motion is continually accelerated by the Action of Gravity, but in a leſs Degree, than when it deſcends perpendicularly; its free Deſcent in this Caſe being hinder'd by the Interpoſition of the Plane: From whence it follows, that what was ſaid in the laſt Chapter, concerning the perpendicular Deſcent of Bodies, is true of ſuch as fall on oblique Planes, Allowance being made for the Difference of Acceleration.

II. The Effect Gravity has upon a Body falling down an oblique Plane, is to that which it exerts upon another falling freely, as the perpendicular Height of the Plane is to its Length †.

III. THE

* See *Keil*'s Introd. ad Phyſ. Præl. 11. *Graveſande* L. 1. Ch. 17.

† *Dem.* Let *AC* (*Fig.* 6.) be the inclin'd Plane, the Body at *A*, and the Action of Gravity, whereby it endeavours to fall, per-

III. The Space, through which a Body falls down the oblique Side of a Plane, is to that through which it would fall perpendicularly in the ſame Time, as the perpendicular Height of the Plane is to its Length *.

For the Space, a Body falls through in any determinate Time, whether down an inclined Plane, or not, is as the Effect of the Gravity with which it is acted upon during that Time; but the Gravity, with which a Body deſcends down the oblique Side of a Plane (by the laſt Propoſition) is to that with which it falls perpendicularly, as the perpendicular Height of the Plane is to its Length: The Space therefore, which a Body falls through obliquely, is to that which it would paſs through perpendicularly in the ſame Time, alſo in that Proportion.

perpendicularly repreſented by the Line *AB*; let *AD* be perpendicular to *AC*, *AD* will then repreſent the Direction by which the Plane acts upon the Body (for all Bodies act in Lines perpendicular to their Surfaces) let then thoſe two Forces be reſolved into one in the Direction *AC*, (as ſhewn in Note to §. 4. Chap. IV.) by compleating the Parallelogram *BD*, whoſe Diagonal will be *AG*. In order to this *BG* muſt be let fall perpendicularly upon *AC* (that it may be parallel to the oppoſite Side of the Parallelogram *AD*) conſequently (8 *Elem.* 6.) *AG* is to *AB* as *AB* to *AC*, that is, the Tendency of the Body down the Plane is to its perpendicular Tendency, as *AB* is to *AC*. Q. E. D.

* From this Propoſition it follows, that ſuppoſing *BG* (*Fig.* 6.) perpendicular to *AC*) the Body would fall from *A* to *G*, in the ſame Time another would fall from thence to *B*; for, as was obſerved (Note the laſt) *AG* is to *AB*, as *AB* to *AC*.

IV. The Velocity, a Body acquires by falling perpendicularly, is to that, which it acquires by falling obliquely in the same Time, as the Space of its perpendicular Descent is to that of its oblique one *.

V. The Time, in which a Body descends through the oblique Side of a Plane, is to that in which it falls through the perpendicular Height of the same, as the Length of the oblique Side is to its Height †.

VI. A Body acquires the same Velocity in falling down the oblique Side of a Plane, as it

* Since by the Note to Section the last, a Body falls to *G*, (*Fig.* 6.) in the same Time another falls to *B*, and by (Chap. V. §. 7.) the Space, a falling Body passes over in any Time, is half that which it would run over in the same Time moving uniformly with its last acquir'd Velocity, it follows that the Body falling down the oblique Plane would pass over double the Space *AG*, moving uniformly with its last acquir'd Velocity, in a Portion of Time equal to that in which it was acquir'd; likewise double the Space *AB* would be passed over by the other Body, moving uniformly with its last acquir'd Velocity, in a Portion of Time equal to that in which it was acquir'd; but since the Velocities of Bodies moving uniformly are as the Spaces they run over in equal Times, the Velocities of the Bodies in *G* and *B* are to each other as double the Lines *AG* and *AB*, that is, as the Lines themselves, which by §. 3. are as the Spaces run through in the same Time, from whence the Proposition is clear.

† *Dem.* The Square of the Time in which *AC* (*Fig.* 6.) is run over, is to the Square of the Time in which *AG* is run over as *AC* to *AG*, (by Chap. V. §. 4) that is, since *AC*, *AB*, *AG* are continually proportional (8 *Elem.* 6.) as the Square of *AC* to the Square of *AB* (by *Def.* 10. *Elem.* 5.) therefore the Times themselves are as the Lines *AC* and *AB*, that is, as the oblique Side of the Plane to the perpendicular Height. *Q. E. D.*

it would do, if it fell freely through the perpendicular Height of it *.

VII. A Body takes up the ſame Time in falling through the Chord of a Circle, whether it be long or ſhort, as it does in falling perpendicularly through the Diameter of the ſame Circle †.

VIII. Upon this is founded the Theory of Pendulums: For from hence it follows, that ſuppoſing a Pendulum could be made to vibrate in a Chord of a Circle, inſtead of an Arch, all its Vibrations would require the ſame Time, whether they were large or ſmall ||.

IX. From hence we ſee the Reaſon, why the ſhorter Arches a Pendulum deſcribes, the

* *Dem.* The Square of the Velocity which a Body acquires by falling to *G*, is to the Square of the Velocity it acquires by falling to *C*, as the Space *AG* to the Space *AC* (by Chap. V. §. 4.) that is (by 8. *Elem.* 6. and *Def.* 10. *Elem.* 5.) as *AGq* to *ABq*; conſequently the Velocity itſelf at *G* is to the Velocity itſelf at *C*, as *AG* to *AB*: But ſince *AG* is run over in the ſame Time *AB* is (ſee Note to §. 3.) the Velocity in *G* is alſo to the Velocity in *B*, as *AG*, to *AB*, (by §. 4.) and conſequently ſince the Velocities both in *C* and *B* bear the ſame Proportion to that in *G*, they muſt be equal to each other. *Q. E. D.*

† *Dem.* It was demonſtrated (§. 3.) that a Body will fall from *A* to *G*, (*Fig.* 7.) on the inclin'd Plane *AC*, in the ſame Time another would fall freely to *B*, provided *AGB* is a right Angle, in which Caſe *AG* (by 31. *Elem.* 3.) is a Chord of that Circle of which *AB* is the Diameter; therefore a Body falls through the Chord, *&c.* Q. E. D.

|| This may be illuſtrated by conceiving the laſt Figure inverted (as in *Fig.* 8.) where ſuppoſing the Ball ſuſpended in ſuch a Manner, as to ſwing in the right Line *GA* inſtead of the Arch *GA*, it would always fall through it in the ſame Time, however long or ſhort it was, for the Inclination of the Line *GA* to the horizontal Line *BC*, is not alter'd by inverting the Figure.

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nearer its Vibrations come to an Equality, for ſmall Arches differ leſs from their Chords than large ones. But if the Pendulum is made to vibrate in a Curve, which Mathematicians call a *Cycloid*; each Swing will then be perform'd in the ſame Time, whether the Pendulum moves through a larger or leſſer Space. For the Nature of this Curve is ſuch, that the Tendency of a Pendulum towards the loweſt Point of it, is always in Proportion to its Diſtance from thence; and conſequently let that Diſtance be more or leſs, it will always be run over by the Pendulum in the ſame Time *.

X. The Time of the Deſcent and Aſcent of a Pendulum, ſuppoſing it to vibrate in the Chord of a Circle, is equal to the Time in which

* The Deſcription of a *Cycloid:*

Upon the right Line *AB*, (*Fig.* 9.) let the Clrcle *CDE* be ſo plac'd, as to touch the Line in the Point *C*, then let this Circle roll along upon it from *C* to *H*, as a Wheel upon the Ground, then will the Point *C* in one Revolution of the Circle deſcribe the Curve *CKH*, which is called a Cycloid. Now ſuppoſe two Plates of Metal bent into the form *HK* and *KC*, and placed in the Situation *LH* and *LC*, in ſuch Manner, that the Points *H* and *C* may be apply'd to *L*, and the Points anſwering to *K* be apply'd to *H* and *C*. This done, if a Pendulum as *LP*, in Length equal to *LH*, be made to vibrate between the Plates or Cheeks of the Cycloid *LC* and *LH*, it will ſwing in the Line *CKH*; and the Time of each Vibration, whether the Pendulum ſwings through a ſmall or a great Part of the Cycloid, will be to the Time a Body takes up in falling perpendicularly through a Space equal to *IK*, (half the Length of the Pendulum:) as the Circumference of a Circle to its Diameter, and conſequently it will always be the ſame

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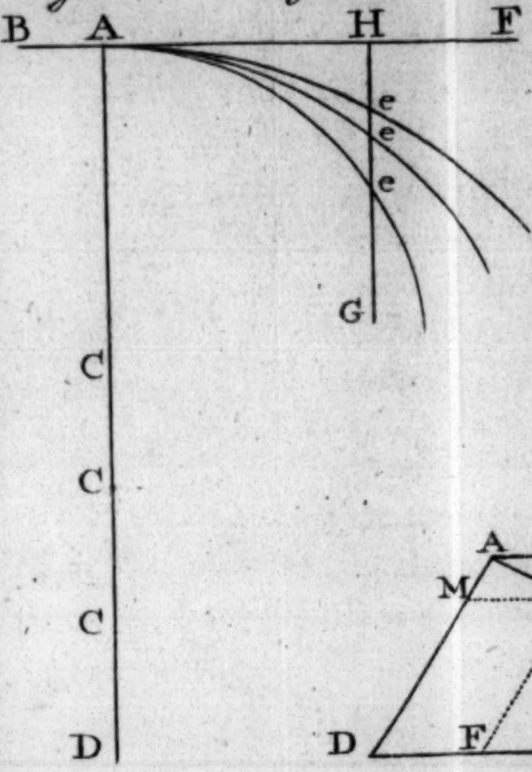
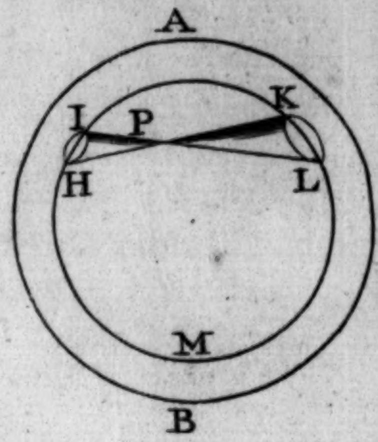
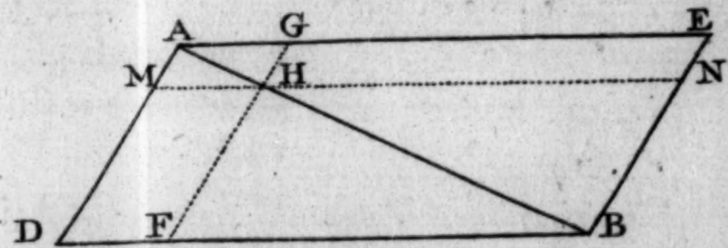
Figure 1. Page 8.



F. 2. P. 16.



F. 3. P. 21.



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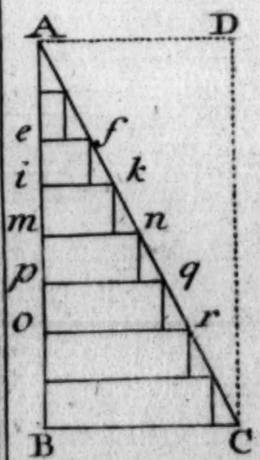
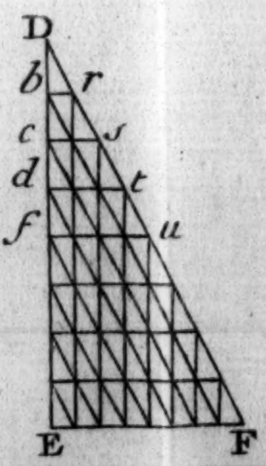
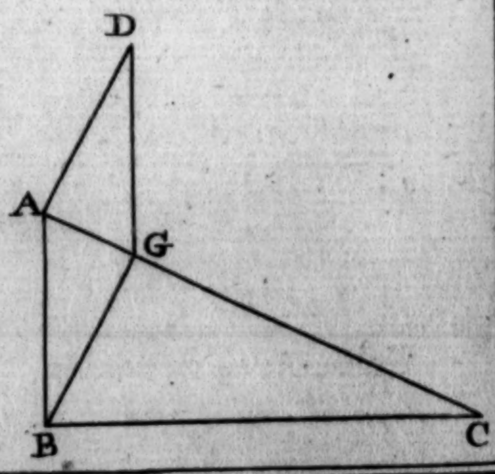


F. 5. P. 26.



F. 6. P. 28.



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which a Body falling freely would descend through eight Times the Length of the Pendulum.

For the Time of the Descent alone upon the Chord is equal to that in which a Body would fall through the Diameter of the Circle (by §. 7.); that is, twice the Length of the Pendulum: But in twice that Time (*viz.* during a whole Vibration) the Body would fall four Times as far (Chap. V. §. 4.), that is, through eight Times the Length of the Pendulum.

XI. The Times, that Pendulums of different Lengths perform their Vibrations in, are as the square Roots of their Lengths *.

XII. The Center of *Oscillation* is a Point in which, if the whole Gravity of a Pendulum was collected, the Time of its Vibration would not be alter'd thereby †; this is the Point from whence

See the Description of a *Cycloid* in the foregoing Page, demonstrated in the Appendix.

* *Dem.* Let there be two Pendulums *A* and *B* (*Fig.* 10. and 11.) of different Lengths, the Time the first vibrates in (suppose through a Chord) is equal to the Time in which a Body would fall freely through *DA*, the Diameter of the Circle (as demonstrated §. 7.); in like Manner the Time *B* vibrates in is that in which a Body would fall through *FB*. Now the Times in which Bodies fall through different Spaces are as the square Roots of those Spaces, that is, of *DA* and *FB*, or of their Halves *CA* and *CB*, *i. e.* of the Lengths of the Pendulums. *Q. E. D.*

† The Rule for finding the Center of Oscillation.

whence the Length of a Pendulum is meaſur'd, which in our Latitude, in a Pendulum that ſwings Seconds, is thirty nine Inches and two Tenths.

XIII. The Squares of the Times in which Pendulums, acted upon by different Degrees of Gravity, perform their Vibrations in, are to each other, inverſly as the Gravities *.

From whence it follows, that a Pendulum will vibrate ſlower when nearer the Equator, than the ſame when nearer the Poles; for the

If the Ball *AB* (*Fig.* 12.) be hung by the String *CD*, whoſe Weight is inconſiderable, the Center of Oſcillation is found thus; ſuppoſe *E* the Center of the Globe, take the Line *K* of ſuch a Length, that it ſhall bear the ſame Proportion to *ED* as *ED* to *EC*, then *EH* being made equal to $\frac{2}{5}$ of *K*, the Point *H* ſhall be the Center of Oſcillation.

If the Weight of the Rod *CD* be too conſiderable to be neglected divide *CD* (*Fig.* 13.) in *I*, ſo that *DI* may be equal to $\frac{1}{3}$ of *CD*, and make a Line as *G*, in the ſame Proportion to *CI*, that the Weight of the Rod bears to that of the Globe, then having found *H* the Center of Oſcillation of the Globe, as before, divide *IH* in *L*, ſo that *IL* may bear the ſame Proportion to *LH*, as the Line *CH* bears to the Line *G*; then will *L* be the Center of Oſcillation of the whole Pendulum. See the Method of finding a general Rule for determining the Center of Oſcillation in all Caſes whatever, in the Appendix.

* *Dem.* The Spaces, falling Bodies deſcend through, are as the Squares of the Times, when the Gravity by which they are impell'd is given (Chap. V. §. 4); and as the Gravity when the Time is given (for the Sum of the Velocities produced in any Time will always be as the generating Forces:) Conſequently when neither is given, they are in a Ratio compounded of both; the Squares of the Times are therefore inverſly as the Gravities. [*For if in* 3 *Quantities* a b, c; a *is as* b c, *then* b: $\frac{a}{c}$, i. e. *if* a *is given, as* $\frac{1}{c}$ *or as* c *inverſly.*] But if the

the Gravity of all Bodies is leſs, the nearer they are to the Equator; *viz.* on account of the ſpheroidical Figure of the Earth, and its Rotation about its Axis, as will be explain'd hereafter. To which we may add the Increaſe of the Length of the Pendulum occaſion'd by the Heat in thoſe Parts; (for we find by Experiment, that Bodies are inlarged in every Dimenſion in Proportion to the Degree of Heat that is given them;) for which Reaſon (§. 11.) the Vibrations of the Pendulum will alſo be ſlower.

CHAP. VII.

Of Projectiles.

A BODY, projected in a Direction parallel or oblique to the Horizon, would proceed on *in infinitum* in a right Line (by the firſt Law of Nature); but being continually accelerated towards the Earth by its Gravity, it will deſcribe a Curve called a *Parabola* *.

the Squares of the Times, in which Bodies fall through given Spaces, are inverſly as the Gravities by which they are acted upon; then the Squares of the Times, in which Pendulums of equal Lengths perform their Vibrations, will be alſo in the ſame Ratio, on account of the conſtant Equality between the Time of the Vibration of a Pendulum, and of the Deſcent of a Body through eight Times its Length (§. 12.)

* *Dem.* Let us ſuppoſe the Body thrown from *A* in the Direction *AB* horizontally (*Fig.* 14.) or obliquely (*Fig.* 15.) it would (if not attracted towards the Earth) move uniformly from

from A towards B, that is, in equal Times it would describe equal Parts of the Line AB, as AC, CD, DE, &c. but, if in the first Portion of Time, while it would move from A to C, it would have descended from A to G by its Gravity, had it only been let drop from thence; it will by a Composition of these two Motions (Chap. IV. §. 2.) at the End of that Time be found in H, the opposite Angle of the Parallelogram $ACGH$. Then in twice that Time, *viz.* while it would have moved over two equal Portions, or from A to D, it would fall downwards to M, four Times as far as before (Chap. V. §. 4.) and will therefore be found in I, supposing DI equal and parallel to AM. Then again in three Portions of Time, or while it would have moved over three Divisions, that is from A to E, it would have fallen downwards nine Times as far as in the first Portion of Time. and therefore being carried by these two Motions will at the End of that Time be found in K, supposing EK, or its equal AN, nine Times as long as AG or CH, &c. Therefore the Lines CH, DI, EK, &c. which are to each other as the Numbers 1, 4, 9, &c. are as the Squares of the Lines AC, AD, AE; (these being only as the Numbers 1, 2, 3.) But this is the Property of the Parabolic Curve. (See De L'*Hospital* B. I. Prop. 1. Corol. 2. and Prop. 3. Corol. 1.) Consequently the Curve $AHIK$, &c. which the Body moves in, whether thrown horizontally or obliquely, is a Parabola. *Q. E. D.*

Lemma 1. The Quotient which arises from the Division of the Square of the Line AC by the Line AG, *viz.* the Quantity $\frac{ACq}{AG}$ (in either of the Parabolic Curves, *Fig.* 14. or 15) or the Square of the Line AD divided by AM, *viz* $\frac{ADq}{AM}$, or the Square of AE divided by AN, viz. $\frac{AEq}{AN}$ is equal to the *Parameter* of the Point A. for GHq is equal to AG multiplied by the *Parameter* (De L'*Hospital* Con. Sect. B. 1. Prop. 1 & 3.) therefore the Parameter is equal to GHq divided by AG, that is, $\frac{ACq}{AG}$. The same is demonstrable of AD divided by AM, &c. and consequently any of these Quantities may be indifferently put to express the Parameter of the same Point.

Lemma 2. The Velocity a Body would acquire by falling from an Height equal to the fourth Part of the Parameter of the Point A, is to the Velocity it would acquire by falling from A to N, as AE is to twice AN. *Dem.*

Dem. Since we are comparing the Velocity, which a Body would acquire by falling through a fourth Part of the Parameter, with that which it would acquire by falling to *N*, let $\frac{AEq}{AN}$ be made choice of to denote the Parameter. Then $\frac{\frac{1}{4}AEq}{AN}$ will express a fourth Part of the Parameter. Now because the Velocities, acquir'd by falling Bodies, are as the square Roots of the Spaces they fall through (Chap. V. §. 6.) the Velocity, acquir'd by a Body in descending through $\frac{\frac{1}{4}AEq}{AN}$ is to that Velocity, which it would acquire by falling through *AN*, as the square Root of $\frac{\frac{1}{4}AEq}{AN}$ to the square Root of *AN*; that is, extracting the Roots of those Quantities as $\frac{\frac{1}{2}AE}{\sqrt{AN}}$ to $\sqrt{AN}$, and, multiplying each Term by $\sqrt{AN}$, as $\frac{1}{2}AE$ to *AN*, or as *AE* to twice *AN*. Q. E. D.

Prop. The Velocity a Body ought to be projected with, to make it describe a given Parabola, is such as it would acquire by falling through a Space equal to the fourth Part of the Parameter belonging to that Point of the Parabola, from whence it is intended to be projected.

Dem. The Velocity with which a Body must be projected from *A* towards *B*, to make it describe the given Parabola *AHIK*, must be such, as would carry it to *C* by an uniform Motion, in the same Time that it would descend by its Gravity from *A* to *G*; and to *E* in the Time it would fall to *N*, &c. as was before observed. Now the Velocity, with which the Line *AE* is described with an uniform Motion, is to that which is acquired by the Body in falling to *N* in the same Time, as *AE* is to twice *AN*; because (Chap. V. §. 7.) its Velocity in *N* would have carried it over twice *AN* in that Time, had it also been uniform. But by *Lemma* 2. the Velocity a Body would acquire, by falling through a Space equal to a fourth Part of the Parameter of the Point *A*, is to that which it would acquire by falling from *A* to *N*, also as *AE* to twice *AN*. Since therefore the Velocity, with which the Line *AE* is described (or, which is the same Thing, that whereby the Body is projected) and that which a Body would acquire by falling through a fourth Part of the Parameter of the Point *A*,

A, bear one and the same Proportion to that Velocity which a Body would acquire by falling from *A* to *N*, they must be equal. *Q. E. D.*

Corol. This affords us an easy Method of finding what Direction it is necessary to throw a Ball in with a given Velocity, in order to strike an Object in a given Situation, *v. g.* Let it be requir'd to strike an Object as *K*, with a Ball thrown from *A* with a given Velocity. Here it is only necessary to make the Triangle *ANK* (suppose a right Line drawn from *A* to *K*) such that $\frac{NKq}{AN}$, or which is the same Thing $\frac{AEq}{EK}$ in the Triangle *AEK*, may be equal to four Times the Space a Body must fall through, to acquire such a Degree of Velocity as that with which it is intended to be thrown, and then *AE* will be the Direction sought. In order to this we must lay down the following Lemma.

Lemma. Let there be a Circle as *ABC* (*Fig.* 16) *AK* a Tangent in the Point *A*, *AB* and *KI* parallel to each other, and let the other Lines be drawn, as in the Figure, I say $\frac{AEq}{EK} = AB$. For the Angle *ABE* is equal to the Angle *EAK* (32. *Elem.* 3.) and the Angle *BAE* is equal to the Angle *AEK* as alternate, therefore the Triangles *ABE* and *AEK* are similar; consequently *AB* is to *AE*, as *AE* to *EK*, and multiplying the extreme Terms together, and middle Terms together, $AB \times EK = AEq$ and dividing both Sides of the Equation by *EK*, $AB = \frac{AEq}{EK}$. Q. E, D. By the same Method of arguing $\frac{AIq}{IK}$ may be proved equal to *AB*.

The Problem.

Let it be requir'd to strike an Object in a given Situation as *K* (*Fig.* 17.) with a Ball projected from *A* with a given Velocity.

Solution. Erect *AB* perpendicular to the Horizon, and equal to four Times the Height a Body must fall from, to acquire the Velocity with which the Ball is to be thrown; bissect this in the Point *G*, through which draw *HC* perpendicular to *AB*, and meeting the Line *AC* (perpendicular to *AK*) in *C*. On *C* as a Center with the Radius *CA*, describe the Circle *ABD*; lastly, through *K* draw the Line *KEI* perpendicular

II. THE greateſt horizontal Diſtance, to which a Body can be thrown with a given Velocity, is at the Elevation of 45 Degrees*.

III. IF two Balls are thrown at different Elevations (but with equal Degrees of Velocity) the one as much above forty five Degrees as the other below, the horizontal Diſtances (or Randoms) where they both fall, will be the ſame †.

IV. THE

pendicular to the Horizon, cutting the Circle in the Points *E* and *I*; I ſay *AE* or *AI* will be the Direction ſought.

For by the Lemma, $AB = \frac{AEq}{EK}$ or $\frac{AIq}{IK}$, but (*ex conſtructione*) *AB* is equal to four Times the Height a Body muſt fall from, to acquire the Velocity with which it is to be thrown, therefore its Equal $\frac{AEq}{EK}$ or $\frac{AIq}{IK}$, is the ſame, which by the Corollary was the Thing requir'd to determine the Direction ſought; conſequently the Parabola, which the Body will deſcribe, will paſs through the Point *K*. Q. E. D.

Coroll. 1. From hence it is evident, that if the Object to be ſtruck be placed any where in the horizontal Line *AO* (*Fig.* 18.) beyond *Q*, the Problem is impoſſible; for then *QH* will not touch the Circle, and the Ball will not reach that Point with any Direction whatever.

* And that when the Ball is directed towards *H*, it will fall on *Q* the greateſt Diſtance it can poſſibly be thrown to; but the Angle *QAH* being equal to *ABH* in the oppoſite Segment (32. *Elem.* 3.) is equal to half *AGH* at the Center (20 *Elem.* 3.) which is a right one; conſequently *QAH* is an Angle of 45 Degrees.

† *Coroll.* 2. If the Object is ſituated in the horizontal Line *AO* (*Fig.* 19) but nearer to *A*, than the greateſt horizontal Diſtance at which it may be ſtruck, ſuppoſe in *K*; the two Directions *AE* and *AI*, with which it may be hit, are equally diſta

IV. The Height a Body will riſe to, when thrown perpendicularly upwards, is equal to half the greateſt horizontal Diſtance it can be thrown to, with the ſame Velocity*.

From hence we may eaſily know how far a Mortar-Piece, or other ſuch Machine, will carry a Ball. Let the Ball be ſhot perpendicularly upwards, note the Time of its Aſcent and Deſcent, half that is the Time of Deſcent, from whence we learn the Height from which it falls (for Bodies are obſerv'd to fall in the firſt Second of Time ſixteen Feet, conſequently in two Seconds they fall four Times ſixteen Feet (Chap. V, §. 4.) in three, nine Times as much, &c. but the perpendicular Height from whence it falls is the ſame with that to which it aſcended, conſequently (§. 4.) the double of this is equal to the greateſt horizontal Diſtance to which that Machine will carry the Ball with an equal Charge.

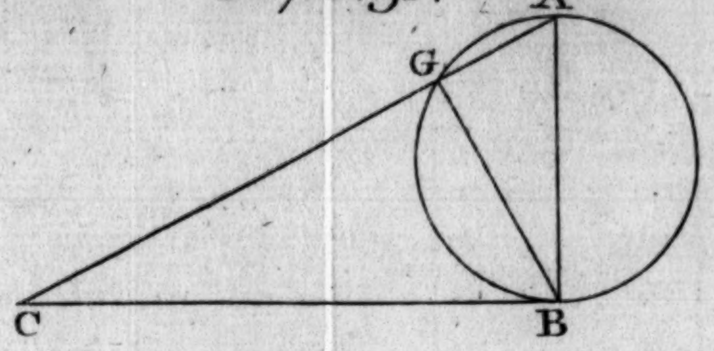
diſtant from the Direction *AH*; for the Angles *IAH* and *HAE* are equal, as inſiſting on equal Arches *IH* and *HE*, (28. *Elem.* 3.)

* *Coroll.* 3. The Altitude of a perpendicular Projection is equal to a fourth Part of the Height *AB*; for the Velocity, with which the Body is projected. is (*ex hypoth.*) ſuch as it would acquire by falling through a fourth Part of the Line *AB*: but a fourth Part of the Line *AB* is equal to half the Line *GH*, or *AQ* (*Fig.* 18.) that is, half the greateſt horizontal Diſtance to which the Body can be thrown.

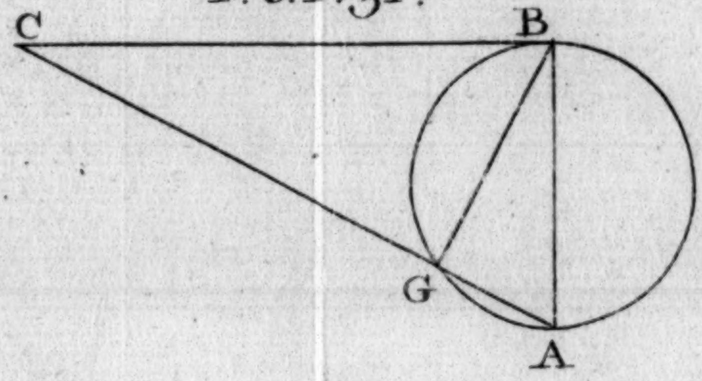
See *Cotes*'s Harmonia Menſurarum, p. 87. *Keil*'s Introduct. ad Phyſ. Præl. 16.

V. The

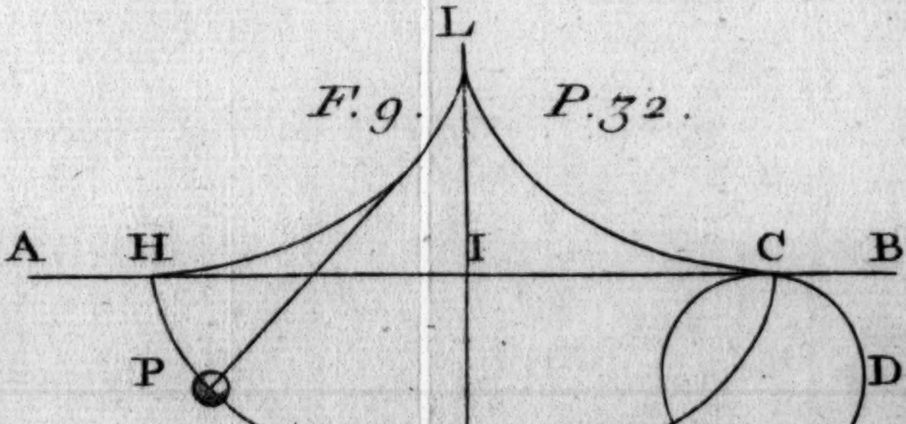
F. 7. P. 31.



F. 8. P. 31.



F. 9. P. 32.



V. The Randoms of two Projectiles, having the ſame Degrees of Elevation, but thrown with different Velocities, are as the Squares of the Velocities: For by the laſt, the Randoms are equal to double the Heights to which the Bodies thrown perpendicularly upwards will aſcend, but the Heights are (Chap. V. §. 6.) as the Squares of the Velocities, therefore the Randoms are ſo too.

VI. Supposing the Motion of the Earth, all Bodies, when thrown perpendicularly upwards, deſcribe *Parabola*'s; notwithſtanding they appear both to aſcend and deſcend in the ſame right Line.

This may very eaſily be illuſtrated in the following Manner; let there be a Body carried uniformly along the Line AB (*Fig.* 20.) by the Motion of the Earth from A towards B; as it paſſes the Point C let it be projected upwards, by ſome Force acting underneath it in the Direction CO perpendicular to the former; the Body will not thereby loſe its Motion, which it had in common with the Earth, towards B (by the firſt Law of Nature) but will be carried by two Motions, one towards B, the other towards O; let us then ſuppoſe, that in the Time it would have advanced forwards to P in the Line AB, it riſes upwards to M in the Line CO; it will then be found in D (Chap. IV. §. 2.) In like Manner, ſuppoſing it would have advanced forward to Q

while it riſes to N, it would then be found in E, afterwards in F, then in G, *&c.* deſcribing the Curve CGL which (from what was demonſtrated under §. 1.) is a Parabola*.

The Reaſon, why it appears to a Spectator to riſe and fall perpendicularly, is becauſe he is carried uniformly along with it by the Motion of the Earth in the Direction AB. *v. g.* Suppoſe the Spectator at C at the Inſtant the Body is thrown from thence, when it arrives at D, he will be moved to P, when the Body is at E he will be at Q, *&c.* as is evident from what was obſerved about the Motion of the Body in the Curve; and they will both meet in L. Therefore ſince the Spectator imagines himſelf ſtanding ſtill, and ſees the Body always perpendicularly over his Head, he muſt of Courſe think, that it riſes right up, and falls right down.

It may be proper to obſerve here, that Experiments, relating to the Motion of projected Bodies, do not exactly anſwer the Theory, the Reſiſtance of the Air deſtroying Part of their Motion; for which a ſmall Allowance is to be made.

* *Dem.* Suppoſe the Motion the Body had in common with the Earth towards *B* (*Fig.* 21.) and that with which it is projected towards *O*, ſuch, as being compounded (Chap. IV. §. 2.) would have produced a Motion in the Direction *CX*; it will follow from thence, that the Path deſcribed by it will be the ſame, as if it had been thrown in that Direction from a Point as *C* at reſt; but in that Caſe it would have deſcribed a Parabola as *CGL* (§. 1.) therefore alſo in this. *Q. E. D.*

CHAP.

CHAP. VIII.

Of Centripetal and Centrifugal Forces.

WHEN a Body is projected in an horizontal Direction, and by its Gravity made to describe a Parabola, as demonstrated Chapter the last; the Curvature of that Parabola will vary in Proportion to the Velocity with which the Body is thrown, and the Gravity which impels it towards the Earth. For the less its Gravity is in Proportion to the Quantity of Matter it contains, or the greater the Velocity is with which it is projected; the less it will deviate from a straight Line, and the further it will go, before it falls to the Earth. For Instance, if a Bullet be shot out of a Cannon from the Top of a Mountain with a given Velocity in an horizontal Direction, and goes in a Curve Line, suppose to the distance of two Miles from the Foot of the Mountain before it falls to the Ground; the same Bullet, shot with a much greater Velocity, would fly to a much greater Distance before its Fall. And by encreasing the Velocity, the Distance to which it is projected may be encreased as much as you please; so that it will not fall to the Ground, till it is arrived at the Distance of ten, or thirty, or ninety

Degrees; or till it has even ſurrounded the whole Earth, and arrives at the very Top of the Mountain from whence it was projected: In this Caſe it will perform a ſecond Revolution, and ſo on *in infinitum* without a new Projection, provided the Reſiſtance of the Air is taken away. Nay it may be projected with ſuch Violence, that it will continually recede from the Earth, moving in a Curve, till at length it goes out of the Sphere of the Earth's Attraction; after which it will go on in a ſtraight Line without ever returning. Which may thus be illuſtrated.

LET ABC (*Fig.* 22.) repreſent the Earth, M the Top of the Mountain from whence the Body is projected in the Direction MQ: It may be thrown with ſuch Force as to carry it to B before it falls, or to C, or even to go round to M, deſcribing the Circle MDM; or laſtly, it may be made to deſcribe the Curve MO, till it gets out of the Sphere of the Earth's Attraction, ſuppoſe at O, going on afterwards in the infinite ſtraight Line OX, there being nothing to ſtop or alter its Courſe. Farther it may be projected with ſuch a Force from M (*Fig.* 23.) as will cauſe it continually to recede from the Earth, till it arrives at the oppoſite Point G, deſcribing the Curve MKG; and if the Point G is within the Sphere of the Earth's Attraction, the Body will return to M, deſcribing the Curve GLM exactly ſi-

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milar to MKG; and in moving nearer and nearer to the Earth, till it comes to M, will regain what Velocity it loſt in going from M to G, its Gravity conſpiring with its Motion from G to M in the ſame Degree in which it oppoſed it from M to G; conſequently the Body when at M, having recovered the Velocity with which it ſet out, will be enabled to perform a ſecond Revolution in the ſame Curve as before; and ſo on.

AGAIN, ſuppoſe it had been projected from the Point M, with a leſs Degree of Force than would have carried it round in the Circle MDM (*Fig.* 22.) but greater than would have ſuffered it to have fallen to the Earth at the oppoſite Point F (*Fig.* 23.) it would alſo in this Caſe have arrived at the Point of M from whence it ſet out; for the Exceſs of Velocity it would have gained in F, by its Tendency towards the Earth in its Way thither, over and above that, with which it was projected from M; would be ſufficient to carry it off again from the Earth, till it arrived at M; and to make it deſcribe the Path FPM exactly ſimilar and equal to the former, loſing in its Way from F to M juſt ſo much Velocity, as it gained by paſſing from M to F; and thereby it would be inabled to perform an infinite Number of Revolutions in the ſame Curve, without requiring a ſecond Projection.

FROM

From hence it follows, that ſuppoſing a Body projected from a Point at any Diſtance within the Sphere of the Earth's Attraction, with a Force ſufficient to carry it half round without falling to the Surface, it is impoſſible it ſhould fall upon any Part of the other half; but will return to the Point from whence it ſet out, making continual ſucceſſive Revolutions in the ſame Curve; provided it meets with no Reſiſtance from the Medium through which it paſſes, nor any other Obſtacle to obſtruct its Motion *.

From hence alſo it is clear, that, the nearer the revolving Body approaches to the Earth, the faſter it moves; its Velocity being continually increaſed during the Time of its Acceſs towards the Earth, and as much retarded during its Receſs from it. And this Acceleration and Retardation will always be ſuch, that the Body will deſcribe equal Areas in equal Times: The meaning of which is, that if we imagine a Line conſtantly extended from the Center of the Earth to the Center of the Body, that Line will always deſcribe or paſs through equal Surfaces or Spaces in equal

* Gravity is here ſuppoſed to be inverſly as the Squares of the Diſtances from the Earth, for 'tis poſſible that the Force, by which a Body tends towards another, may vary in ſuch a Manner at different Diſtances, that the projected Body ſhall deſcribe a ſpiral Line, continually approaching to, or receding from that about which it revolves.

Times,

Times, for it conſtantly becomes ſhorter the faſter it moves, and *vice verſâ* *.

AND for the ſame Reaſon that a Body, projected with a ſufficient Velocity, may by the Force of Gravity be made to deſcribe a Curve round the Earth, and perform continual ſucceſſive Revolutions therein; it follows, that the Moon may by the ſame Force of Gravi-

* *Dem.* Let the Time in which the Body performs one Revolution be divided into equal Parts, in the firſt of which let the Body deſcribe the right Line *AB* (*Fig.* 24.) in the ſecond Part of Time, if not prevented, it would go ſtraight on to *c*, deſcribing the Line *Bc* equal to *AB* by the firſt Law of Nature; the Lines *SA*, *SB*, *Sc* being drawn, the Triangles *SBA*, *ScB*, will be equal to each other, their Baſes *AB* and *Bc* being equal, and their Heights *S* the ſame (38. *Elem.* 1.) When the Body arrives at *B*, let the centripetal Force by one ſingle Impulſe turn it out of the ſtraight Line *Bc* into the Line *BC*; in which let it move on uniformly without receiving a ſecond Impulſe till it comes to *C*. Let *Cc* be drawn parallel to *SB* meeting *BC* in *C*; then at the End of the ſecond Part of Time, the Body will be found in *C*, having deſcribed the Diagonal of the Parallelogram *Nc* (Chap. IV. §. 2.) Draw *SC*, and the Triangle *SCB* will be equal to the Triangle *ScB*, each having the ſame Baſe *SB* and being between the ſame Parallels *Cc* and *SB*) and therefore alſo equal to the Triangle *SBA*. For the ſame Reaſon, if the centripetal Force acts in the Points *C*, *D*, *E* ſucceſſively, ſo as to make the Body deſcribe the ſtraight Lines *CD*, *DE*, *EF*, &c. in ſo many equal Parts of Time, the Triangles *SCD*, *SDE*, *SEF*, &c. will be all equal to one another and to the Triangle *SAB*. Conſequently equal Areas are deſcribed in equal Times. Let us then ſuppoſe the Baſes of thoſe Triangles, *viz.* *AB*, *BC*, *CD*, *DE*, &c. diminiſhed *in infinitum*, and likewiſe the Times in which they are deſcribed; then will the Perimeter *A*, *B*, *C*, *D*, *E*, *F*, &c. become a Curve, and any Number of thoſe Triangles taken together (or their Areas) will be proportionable to the Times in which they are deſcribed. *Q. E. D.*

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ty be made to revolve about the Earth or any other Planet by the like Force, about the Sun; if the Velocities with which they move are duly adjusted to the Forces, by which they are acted upon.

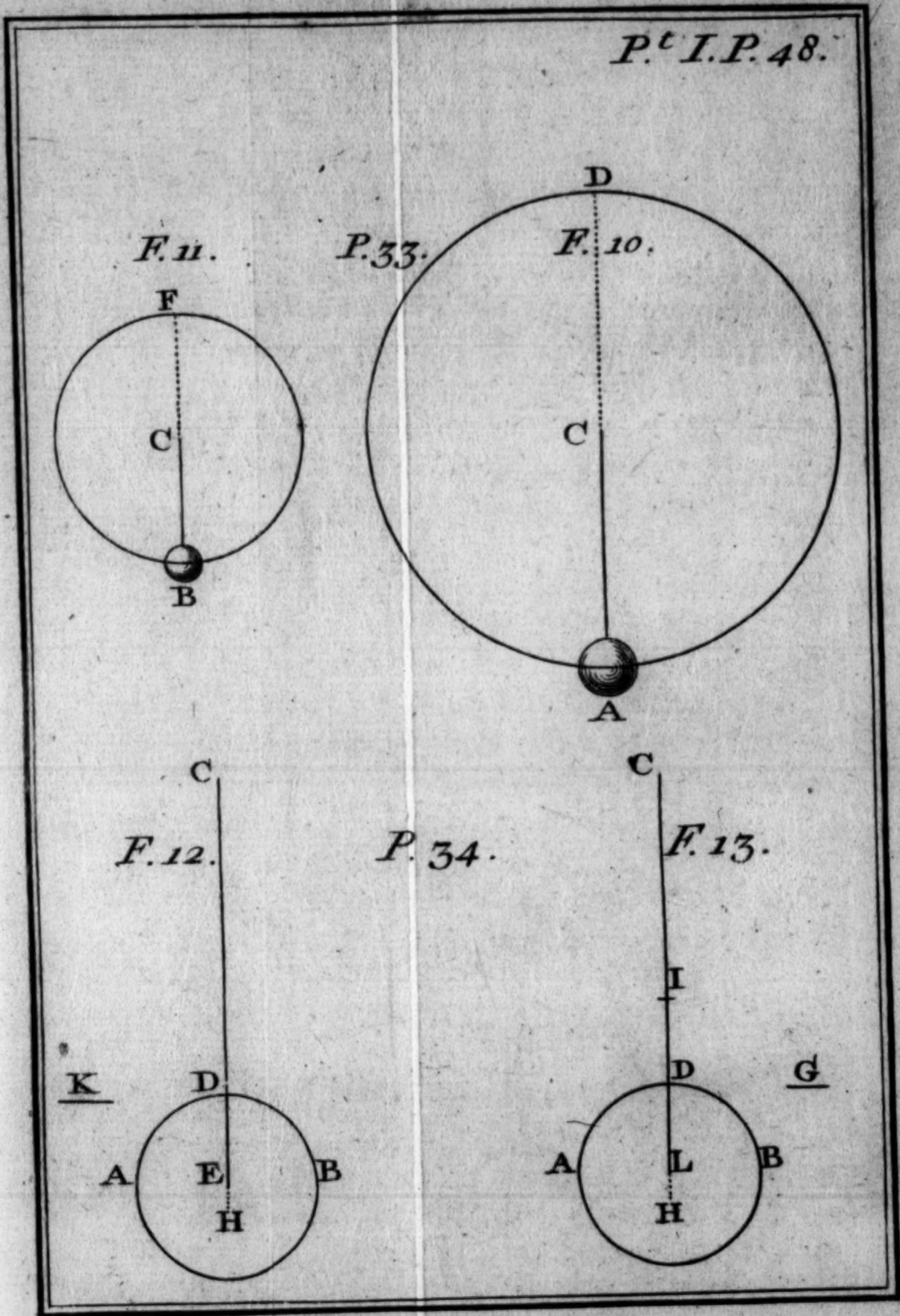
When a Body revolves about another in this Manner, that Force or Power by which it is prevented from flying off (as it otherwise would do in a Tangent to the Curve which it describes) is call'd the *Centripetal*; the Counter-action of this, by which it endeavours to fly off, the *Centrifugal*; these, by the third Law of Nature being equal to each other, are called by one common Name, *Central Forces*; that with which the Body is at first projected, or continues its Motion from any Point, is the *Projectile* Force; and the Time in which it performs one Revolution, the *Periodical* Time.

These Forces, properly relating to the Motions of the Heavenly Bodies, will be more largely treated of in another Place.

CHAP IX.

Of the Communication of Motion.

I. BEFORE we proceed to explain the Laws, by which Bodies communicate their Motion from one to another, it is very necessary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necessary to make a Distinction between Motion and Velocity; which ought to be well observ'd, and is as follows.

By the Motion of a Body (sometimes called its Quantity of Motion, sometimes its *Momentum*) is not to be understood the Velocity only, with which the Body moves; but the Sum of the Motion of all its Parts taken together: Consequently the more Matter any Body contains, the greater will be its Motion, though its Velocity remains the same. Thus, supposing two Bodies, one containing ten Times the Quantity of Matter the other does, moving with equal Velocity; the greater Body is said to have ten Times the Motion, or Momentum, that the other has: For 'tis evident, that a tenth Part of the larger has as much, as the other whole Body. In short, that Quality in moving Bodies, which Philosophers understand by the Term Momentum or Motion, is no other than what is vulgarly called their *Force*, which every one knows to depend on their Quantity of Matter, as well as their Velocity. This is that Power, a moving Body has to affect another in all Actions that arise from its Motion, and is therefore a fundamental Principle in Mechanics.

II. Now, since this Momentum, or Force, depends equally on the Quantity of Matter a Body contains, and on the Velocity with which it moves; the Method, to determine how

great it is, is to multiply one by the other. Thus, ſuppoſe two Bodies, the firſt having twice the Quantity of Matter, and thrice the Velocity which the other has; any two Numbers, that are to each other as two to one, will expreſs their Quantities of Matter (it being only their relative Velocities and Quantities of Matter which we need conſider) and any two Numbers that are as three to one their Velocities; now multiplying the Quantity of Matter in the firſt, *viz.* two by its Velocity three, the Product is ſix; and multiplying the Quantity of Matter in the ſecond by its Velocity, *viz.* one by one, the Product is one; their relative Forces therefore or Powers will be as ſix to one, or the Moment of one is ſix Times greater than that of the other. Again if their Quantities of Matter had been as three to eight, and their Velocities as two to three, then would their Moments have been as ſix to twenty four, that is, as one to four.

THIS being rightly apprehended, what follows, concerning the Laws of the Communication of Motion by Impulſe, and the mechanical Powers, will be eaſily underſtood.

The Communication of Motion.

I. In Bodies not Elaſtic.

III. THOSE Bodies are ſaid to be not *Elaſtic*, which, when they ſtrike againſt one another,

ther, do not rebound, but accompany one another after Impact, as if they were joined. This proceeds from their retaining the Impression, made upon their Surfaces, after the impressing Force ceases to act. For all rebounding is occasioned by a certain Spring in the Surfaces of Bodies, whereby those Parts, which receive the Impression made by the Stroke, immediately spring back, and throw off the impinging Body; now, this being wanting in Bodies void of Elasticity, there follows no Separation after Impact.

IV. When one Body impinges on another which is at rest, or moving with less Velocity the same Way, the Quantity of the Motion or Momentum in both Bodies taken together remains the same after Impact, as before; for by the third Law of Nature, the Reaction of one being equal to the Action of the other, what one gains, the other must lose.

Thus, suppose two equal Bodies, one impinging with twelve Degrees of Velocity on the other at rest: The Quantities of Matter in the Bodies being equal, their Moments and Velocities are the same; the Sum in both twelve; this remains the same after Impact (§. 4.) and is equally divided between them (§. 3.) they have therefore six a-piece, that is, the impinging Body communicates half its Velocity, and keeps half.

V. When two Bodies impinge on each other by moving contrary Ways, the Quantity of Motion, they retain after Impact, is equal to the Difference of the Motion they had before; for by the third Law of Nature, that, which had the least Motion, will destroy an equal Quantity in the other, after which they will move together with the Remainder, that is the Difference.

Thus for Instance, let there be two equal Bodies moving towards each other, the one with three Degrees of Velocity, the other with five, the Difference of their Moments or Velocities will be two; this remains the same after Impact (§. 5.) and is equally divided between them (§. 3.) they have therefore one a-piece: That is, the Body, which had five Degrees of Velocity, loses three or as much as the other had, communicates half the Remainder, and keeps the other half *.

* From these Positions it is easy to reduce a Theorem, that shall shew the Velocity of Bodies after Impact in all Cases whatever. Let there be two Bodies A and B, the Velocity of the first a, of the other b; then (§. 2.) the Moment of A will be expressed by Aa, and of B by Bb; therefore the Sum of both will be $Aa+Bb$; and $Aa-Bb$ will be the Difference when they meet. Now these Quantities (by §. 4. and 5.) remain the same after Impact; but knowing the Quantities of Motion and Quantities of Matter, we have the Velocity (which §. 3. is the same in both) by dividing the former by the latter (as follows from §. 2.) therefore $\frac{Aa+Bb}{A+B}$ or $\frac{Aa-Bb}{A+B}$ will in all Cases express the Velocity of the Bodies after Impact.

II. In

II. In Elaſtic Bodies.

VI. Bodies perfectly *Elaſtic* are ſuch as rebound after Impact with a Force equal to that with which they impinge upon one another: Thoſe Parts of their Surfaces, that receive the Impreſſion, immediately ſpringing back, and throwing off the impinging Bodies with a Force equal to that of Impact.

VII. From hence it follows, that the Action of Elaſtic Bodies on each other (that of the Spring being equal to that of the Stroke) is twice as much as the ſame in Bodies void of Elaſticity. Therefore, when Elaſtic Bodies impinge on each other, the one loſes, and the other gains twice as much Motion as if they had not been Elaſtic; we have therefore an eaſy Way of determining the Change of Motion in Elaſtic Bodies, knowing firſt what it would have been in the ſame Circumſtances, had the Bodies been void of Elaſticity.

Thus, if there be two equal and Elaſtic Bodies, the one in Motion with twelve Degrees of Velocity impinging on the other at reſt, the impinging Body will communicate twice as much Velocity as if it had not been Elaſtic, that is (by §. 4.) twelve Degrees, or all it had; conſequently it will be at reſt, and the other will move on with the whole Velocity of the former.

VIII. It ſometimes happens, that in Bodies not Elaſtic, the one loſes more than half its Velocity, in which Caſe, ſuppoſing them Elaſtic, it loſes more than all; that is, the Exceſs of what it loſes, above what it has, is negative, or in a contrary Direction; thus, ſuppoſe the Circumſtances of Impact ſuch, that a Body, which has but twelve Degrees of Velocity, loſes ſixteen; the overplus four is to be taken the contrary Way, that is, the Body will rebound with ſour Degrees of Velocity. *v. g.* Let it be required to determine the Velocity of a Body after Impact againſt an immoveable Object. Let us firſt ſuppoſe the Object and Body both void of Elaſticity: 'Tis evident the impinging Body would be ſtopp'd or loſe all its Motion, and communicate none; if they are Elaſtic, it muſt loſe twice as much (by §. 7.) and conſequently will rebound with a Force equal to that of the Stroke.

IX It is ſufficient if only one of the Bodies is Elaſtic, provided the other be infinitely hard; for then the Impreſſion in the Elaſtic Body will be double of what it would have been, had they both been equally Elaſtic: And conſequently the Force, with which they rebound, will be the ſame as if the Impreſſion had been equally divided between the two Bodies.

X. There are no Bodies, that we know of, either perfectly Elaſtic, or infinitely hard; the

the nearer therefore any Bodies approach to Perfection of Elasticity, so much the nearer do the Laws, which they observe in the mutual Communication of their Motion, approach to those we have laid down.

XI. Sir ISAAC NEWTON made Trials with several Bodies, and found that the same Degree of Elasticity always appeared in the same Bodies, with whatever Force they were struck, so that the Elastic Power, in all the Bodies he made Trial upon, exerted itself in one constant Proportion to the compressing Force. He found the Celerity with which Balls of Wool, bound up very compact, receded from each other, to bear nearly the Proportion of five to nine to the Celerity wherewith they met; and in Steel, he found nearly the same Proportion; in Cork the Elasticity was something less, but in Glass much greater; for the Celerity, with which Balls of that Material separated after Percussion, he found to bear the Proportion of fifteen to sixteen to the Celerity wherewith they met*.

XII. WE have hitherto supposed the Direction, in which Bodies impinge upon one another, to be perpendicular to their Surfaces: When it is not so, the Force of Impact will be less, by how much the more that Direction varies from the Perpendicular; for it is manifest that a direct Impulse is the greatest of all

* Newt. Princip. Phil. pag 21.

others

others that can be given with the ſame Degree of Velocity *.

XIII. This is the Caſe, when Bodies impel one another by acting upon their Surfaces; but in Forces, where the Surfaces of Bodies are not concerned, as in Attraction, *&c.* we muſt not conſider the Relation which the Direction of the Force has to the Surface of the Body to be moved, but to the Direction in which it is to be moved by that Force. Here the Force of Action will be leſs, by how much the more theſe two Directions vary from each other †. My Meaning in both Caſes

* The Force of oblique Percuſſion is to that of direct, as the Sine of the Angle of Incidence to the Radius.

Dem. Let there be a Plane as *AD* (*Fig.* 25.) againſt which let a Body impinge in the Point *D* in the Direction *BD*: which Line may be ſuppoſed to expreſs the Force of direct Impulſe, and may be reſolved into two others (Chap. IV. §. 2) *BC* and *BA*; the one parallel, the other perpendicular to the Plane; but that Force which is exerted in a Direction parallel to the Plane can no Way affect it; the Stroke therefore ariſes wholly from the other Force expreſſed by the Line *BA*; but this is to the Line *BD*, as the Sine of the Angle of Incidence *ADB* to the Radius; from whence the Propoſition is clear.

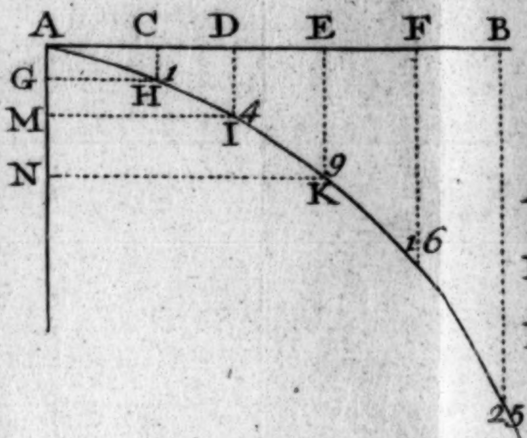
If the Surface of the Body to be ſtruck is a Curve, then let *AD* be made a Tangent to *D* the Point of Incidence, and the Demonſtration will be the ſame.

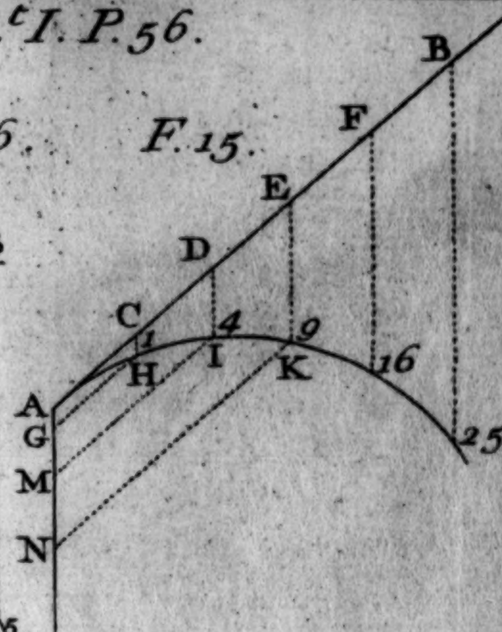
† The Force of oblique Action is to that of direct, as the Co-Sine of the Angle comprehended between the Direction of the Force, and that wherein a Body is to be moved thereby, to the Radius.

Dem. Let *FD* (*Fig.* 26.) repreſent a Force acting upon a Body as *D*, and impelling it towards *E*; but let *DM* be the only Way in which it is poſſible for the Body to move; the Force

Pt. I. P. 56.

F. 14. P. 36. F. 15.





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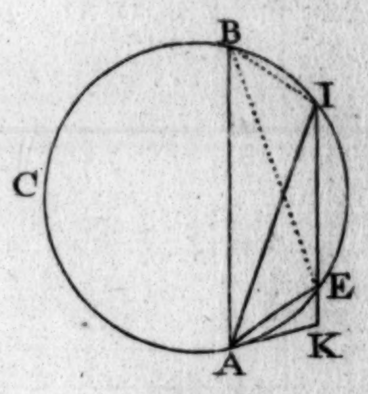

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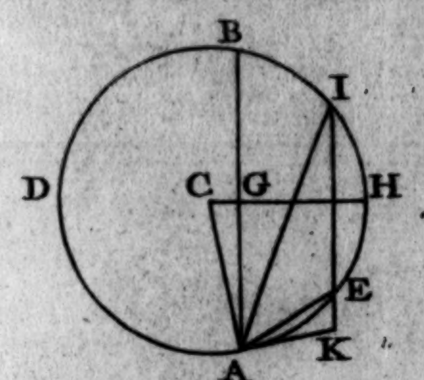

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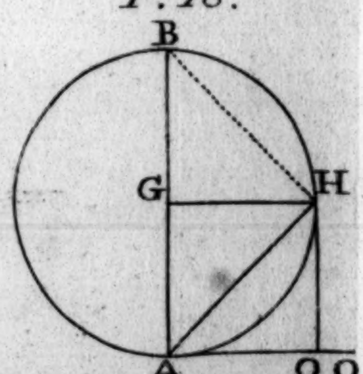

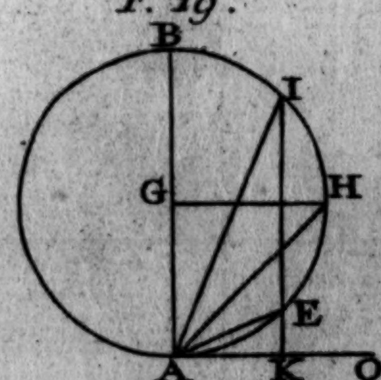

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Caſes will be underſtood from the Inſtance of a Ship under Sail. The Force, by which the Wind acts upon the Sail, will be leſs, by how much the more its Direction varies from one that is perpendicular to its Surface: But the Force of the Sail, to move the Ship forward, will be leſs, by how much the more the Direction of the Ship's Courſe varies from that, in which ſhe is impell'd by the Sail.

XIV. To this we may add the following Propoſition, relating to oblique Forces, *viz.* that, if a Body is drawn or impelled three different Ways at the ſame Time by as many Forces acting in different Directions; and if the Quantity of thoſe Forces is ſuch, that the Body is kept in its Place by them: Then will the Forces be to each other, as the ſeveral Sides of a Triangle, drawn reſpectively parallel to the Directions in which they act *.

Force *FD* may be reſolved (Chap. 4. §. 2.) into two others *FG* and *FH*, or which is equal to *GD*; but 'tis evident that only the Force *GD* impels it towards *M*. Now, *FD* being the Radius, *GD* is the Co-Sine of the Angle *FDG* comprehended between the two Directions *FE* and *GM*; from whence the Propoſition is clear.

* *Dem.* Let the Lines *AB*, *AD*, *AE*, (*Fig.* 27.) repreſent the three Forces acting upon the Body *A* in thoſe Directions, and by that Means keeping it at reſt in the Point *A*. Then the Forces *EA* and *DA* will be equivalent to *BA*, otherwiſe the Body would be put into Motion by them (*contra Hypoth.*) But theſe Forces are alſo equivalent to *AC* (Chap. IV. §. 2.) conſequently *AC* may be made uſe of to expreſs the Force *AB*; and *EC*, which is parallel and equal to *AD*, may expreſs the Force *AD*, while *AE* expreſſes its own: But *ACE* is a Tri-

 angle

CHAP. X.

Of the Mechanical Powers.

I. HAVING, in the foregoing Chapter, accounted for the Communication of Motion by Impulſe; we proceed next to conſider Motion as communicated by Means of certain Inſtruments, commonly known by the Names of *Mechanical Powers*. The Uſe of theſe Powers conſiſts chiefly in managing great Weights, or performing other Works with a determinate Force.

II. THEY are uſually reckoned five. *viz.* The Lever, the Wheel and Axis, the Pully, the Screw, and the Wedge; to which ſome add the inclined Plane. To theſe all Machines how complicated ſoever are reducible.

III. THESE Inſtruments have been of very ancient Uſe; for we find that *Archimedes* was well acquainted with the Extent of their Power; as may be inferred from that celebrated Saying of his, Δὸς πῦ ςῶ, καὶ τὴν γῆν κινήσω. By which he meant, that the greateſt imaginable Weight might be moved with the ſmalleſt Power.

angle whoſe Sides are all parallel to the given Directions; therefore the Sides of this Triangle will expreſs the Relation of the Forces by which the Body is kept at reſt. *Q. E. D.*

IV. THAT

IV. THAT Body, which communicates Motion to another, is called the *Power*; that which receives it, the *Weight*.

V. THAT Point in a Body, which remains at reſt, while the Body is turning round, is called the *Center of Motion*. Beſides this, there are two other Centers in Bodies, 1. That of *Magnitude*, which is a Point, as near as poſſible, equally diſtant from all the external Parts of the Body; 2. That of *Gravity*, or that, about which all the Parts of the Body, in whatever Situation it is placed, exactly balance each other.

VI. WHEN a Body communicates Motion to another, it loſes juſt ſo much of its own, as it communicates to that other; the Action of one being equal to the Reaction of the other. See Chapter the laſt, §. 4. and 5.

VII. WHEN two Bodies have ſuch Relation to each other (ſuppoſe them fixed to different Parts of the ſame Machine) that if one be put into Motion, the other will thereby neceſſarily have ſuch a Degree of Velocity given it, that their Moments will be equal; it will then be impoſſible, that one ſhould begin to move without communicating to the other a Quantity of Motion equal to its own; 'tis evident therefore from the laſt Propoſition, that if we ſuppoſe it to begin to move, in that very Inſtant it muſt loſe all its own Motion by communicating the whole to the other

Body, and therefore, being left to itſelf, will remain at reſt, and communicate none at all.

Now the Moments of two Bodies are equal (Chap. IX. §. 2.) when the Velocity of the firſt is to that of the ſecond, as the Quantity of Matter of the ſecond to that of the firſt; or if we ſuppoſe their Quantities of Matter as one to three, then by the Suppoſition their Velocities are as three to one; and if we multiply the Quantity of Matter in the firſt, *viz.* one, by its Velocity three, and that of the other, *viz.* three, by its Velocity one; their Products are equal; their Moments are therefore by the Definition (Chap. XI. §. 1. and 2.) equal. They will alſo be equal, when the Spaces the Bodies paſs over are in that Proportion; for the Times they both move in being the ſame, the Spaces will always be as the Velocities.

VIII. From hence it follows, that in any Machine, whether ſimple or compound, the Power however ſmall may have a Moment equal to that of the Weight; provided the Machine be ſuch, that when it is in Motion, the Velocity of the Power ſhall bear ſuch Proportion to that of the Weight, as the Weight does to the Power; for then, what the Power wants in Quantity of Matter or Weight, will be made up in Velocity; conſequently their Moments will be equal by §. the laſt, and therefore by §. 7. they will exactly balance each other; or be in *Æquilibrio.*

IX. But

IX. But if the Power bears a greater Proportion to the Weight, than the Velocity of the Weight to that of the Power; it will then have a greater Momentum than the other; ſo that though the other receives all its own Moment from it when the Machine moves, yet there will ſome remain, which if it be ſufficient to overcome the Friction of the Machine, will keep it moving.

We proceed now to treat of each Mechanical Power in its Order, and

I. Of the Lever.

X. The Lever is a right Line (or Bar whoſe Weight in Theory is not conſider'd) moveable on a Center, which is called its *Fulcrum*, or *fixed Point*.

XI. The Æquilibrium in this Machine is, when the Diſtance of the Power from the fixed Point is to that of the Weight from the ſame, as the Quantity of Matter in the Weight is to that in the Power.

For, ſuppoſing the Lever placed on its Fulcrum with the Weight to be raiſed at one End, and the Power applied to the other; 'tis evident, the farther the Power is placed from the Fulcrum or Center of Motion, the larger will be its Sweep when the Machine is put in Motion; that is, it will move over proportionably more Space in the ſame Time than the Weight to be raiſed: now, if it is placed juſt

ſo

ſo many Times farther from the Fulcrum, as it is leſs than the Weight, it will move juſt ſo many Times faſter; their Moments therefore will be equal (§. 7.) and conſequently the Power and Weight will exactly balance each other, or be in Æquilibrio *. And, if the Power is ſufficiently augmented to overcome the Friction of the Machine, it will put it in Motion.

The Lever is of three Kinds. 1. When the fixed Point is between the Weight and the Power, as in the laſt Caſe. 2. When the Weight is between the fixed Point and the Power. 3. When the Power is between the fixed Point and the Weight,

In all which Caſes the Æquilibrium will be, when their Diſtances from the fixed Point are ſuch, that their Velocities ſhall be inverſely as their Quantities of Matter; for then (by §. 7.) being at reſt, neither of them will communicate any Motion to the other

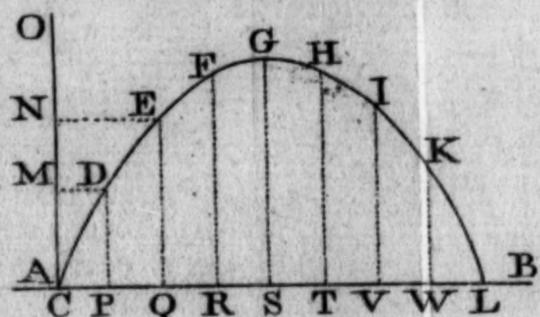
The common Scales may be conſidered as

* Geometrically thus. Let *AB* (*Fig.* 28.) repreſent the Lever, *F* the Fulcrum, *W* the Weight, *P* the Power, the one ſuſpended at the Extremity of the Lever *A*, the other at *B*, and let *BF* be to *FA* as *W* to *P*; then while the Lever moves from the Situation *AB* into that of *CD*, the Point *B* which ſuſtains the Power will move as many Times farther than *A* which ſuſtains the Weight (and conſequently as many Times faſter ſince they perform their Motions in the ſame Time) as the Arch *BD* is longer than *AC*; that is, the Triangles *BFD* and *AFC* being ſimilar, as the Arm *BF* is longer than *AF*, which (*ex Hypoth.*) is as many Times as the Weight exceeds the Power, they will therefore (§. 7.) be in Æquilibrio. *Q. E. D.*

a Lever

P.t I. P. 62.

F. 20. P. 41.



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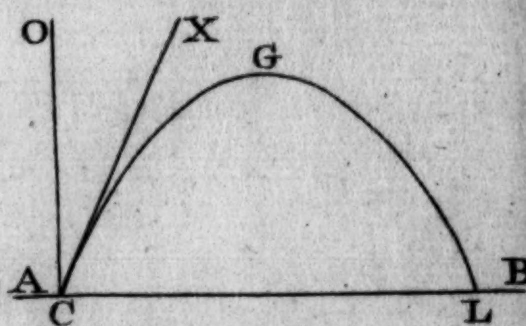
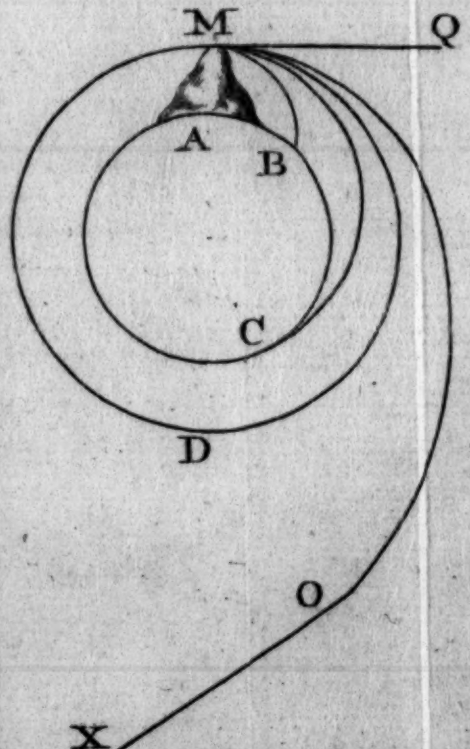
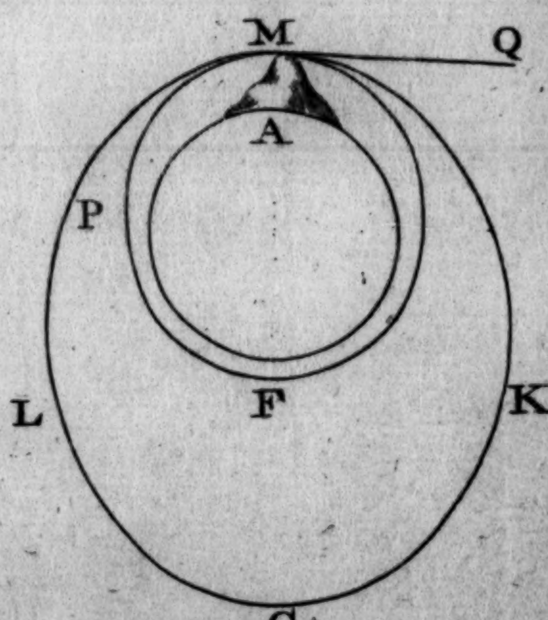


F. 22. P. 44. F. 23.





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a Lever of the firſt Kind, where the Weight and Power are applied at equal Diſtances from the fixed Point.

The Steelyard is alſo a Lever of the firſt Kind, but whoſe Arms are unequal.

THE Difference between the Uſe of the Scales and the Steelyard conſiſts in this; that as in one you make uſe of a larger Power (or more Weights) to eſtimate the Weight of an heavier Body; in the other you uſe the ſame Power, but give it a greater Velocity with reſpect to that of the Weight by applying it farther from the fixed Point, which by §. 7. will have the ſame Effect.

II. The WHEEL and AXIS.

XII. THIS Machine is a Wheel, that turns round together with its Axis; the Power in this is applied to the Circumference of the Wheel, and the Weight drawn up by Means of a Rope wound about the Axis.

XIII. In this there will be an Æquilibrium, when the Weight is to the Power, as the Diameter of the Wheel to the Diameter of the Axis.

'TIS evident, the Velocity of the Power will exceed the Velocity of the Weight, as many Times as the Circumference of the Wheel exceeds that of its Axis; becauſe the Spaces they paſs over in one Revolution will be as thoſe Circumferences; that is, as many Times as the Diameter

meter of one exceeds that of the other (the Circumferences of Circles being as their Diameters;) what therefore in this Caſe the Power wants in Weight will be made up in Velocity, from whence (§. 7.) there will be an Æquilibrium *.

The Uſe of this Machine is to raiſe Weights to greater Heights than the Lever can do, becauſe the Wheel is capable of being turned ſeveral Times round, which the Lever is not; and alſo to communicate Motion from one Part of a Machine to another; accordingly there are few compound Machines without it.

III. The Pulley.

XIV. A Pulley is an Inſtrument compoſed of one or more Wheels moveable on their Axes.

XV. A ſimple Pulley, if its Axis is fixed, is of no other Uſe, than to alter the Direction of the Power; for the Power and Weight will both move through an equal

* Geometrically thus. Let *AB* (*Fig.* 29.) be the Diameter of the Wheel, *DE* that of the Axis, *W* the Weight, and *P* the Power; when the Wheel begins to move, the Point *B* and *D* will deſcribe ſimilar Arches about the Center *C*, in the ſame Manner the Point *A* and *B* in the Lever were ſhewn to do about the fixed Point *F* (*Fig.* 28.) that is, the Point *B* will move as many Times faſter than *D*, as *CB* is longer than *CD* or *AB* than *DE*, the Motion therefore of *P* (§. 7.) will be equal to that of *W*. From whence the Propoſition is clear.

Space

Space in the ſame Time. But in a Pulley not fixed, as in *Fig.* 30. where the Rope runs under it, or in a Combination of Pullies as in *Fig.* 31. the Æquilibrium will be, when the Power is to the Weight, as one to the Number of Ropes, that paſs between the upper and lower Pullies.

For, ſuppoſe one End of the Rope fixed in B (*Fig.* 30.) and the other ſupported by the Power P, it is evident, that in order to raiſe the Weight W one Foot, the Power muſt riſe two, for both Ropes, *viz.* BC and CP, will be ſhortened a Foot a-piece, when the Space run over by the Power will be double to that of the Weight; if therefore the Power is to the Weight as one to two, their Moments will be equal: for the ſame Reaſon, if there be four Ropes paſſing from the upper to the lower Pullies as in *Fig.* 31. the Velocity of the Power will be quadruple to that of the Weight, or as four to one, &c. In all Caſes therefore when the Power is to the Weight, as one to the Number of Ropes paſſing from the upper to the lower Pullies (§. 7.) there will be an Æquilibrium.

XVI. If the Pullies be diſpoſed as in Figure the 32d, each having its own particular Rope, the Action of the Power will be very much increaſed; for here every Pulley doubles it, wherefore the Power is four Times greater with two Pullies, eight Times with three,

ſixteen Times with four, &c. For, it is evident from the Conſideration of the Figure, the firſt will move half as faſt as the Power, the ſecond half as faſt as that, and ſo on; wherefore (§. 7.) the Power is doubled by each Pulley.

The Uſe of the Pulley is nearly the ſame with that of the Wheel and Axis, but it is more portable, and eaſier to be fixed up.

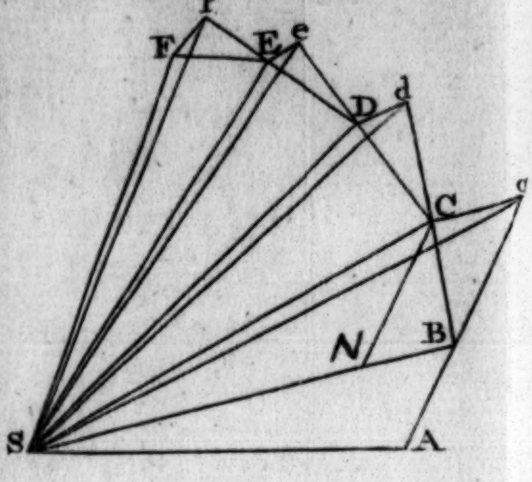
IV. The Screw.

XVII In this Machine the Æquilibrium will be, when the Power is to the Weight, as the Diſtance between any two contiguous Threads or Spirals in the Screw, to the Way deſcribed by the Power in one whole Revolution. It is manifeſt from the Form of the Machine (*Fig.* 33.) that in one Revolution of the Screw, the Weight will be moved through a Space equal to the Diſtance of two contiguous Threads, and that the Power will run through a Space equal to the Compaſs it takes in one Revolution, therefore (§. 7.) if the Weight is to the Power in that Proportion, there will be an Æquilibrium.

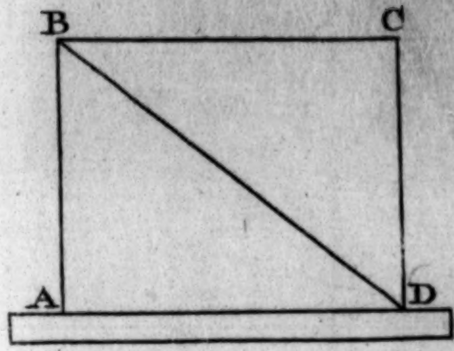
This Machine is of great Force, and very uſeful in retaining Bodies in a compreſſed State, becauſe it will not run back, as the three foregoing will, when the Power is removed. This ariſes from the great Friction of

P.t I. P. 66.

F. 24. P. 47.



F. 25. P 56.



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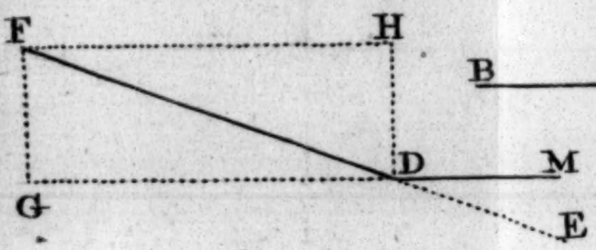

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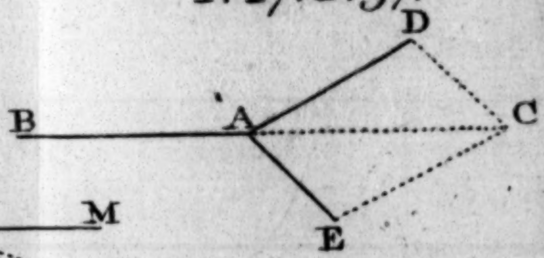

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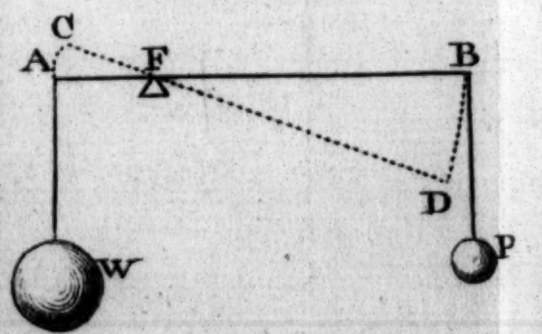
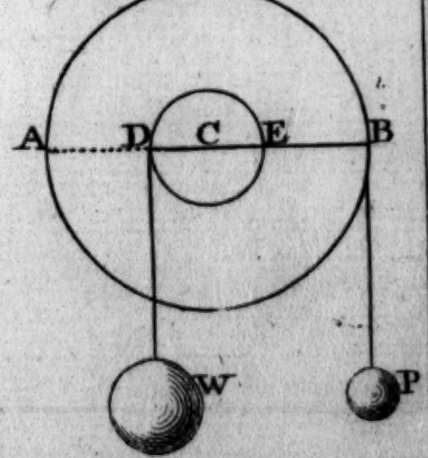


F. 29. P. 64.



of thoſe Parts in the Screw, which during its Motion ſlide upon thoſe that are at Reſt.

V. The WEDGE.

XVIII. THIS Inſtrument is formed by two equal Rectangles, joined at their lower Baſes, and ſeparated at their upper ones, by a third; which is called the *Back* of the *Wedge*; the other two, its *Sides*.

XIX. In the foregoing Mechanical Powers we have all along conſidered the Weight, as moved in the ſame Direction with that, in which it is acted upon by the Machine, as is commonly the Caſe; but in this, the Weight is generally applied in ſuch a Manner, as to be made to move in a Direction different from that, in which it is protruded by the Wedge; hence it is, that Mathematicians have widely differed in their Determination of the Power of this Machine, ſome conſidering the Weight as moved by it in one Direction, and ſome in another. Nay, there are ſome, even among the late Writers, that have been led into manifeſt Errors by it. We will therefore lay down the ſeveral Proportions, they have given us, for the determining the Power of this Machine, and examine them one by one. 1. It is demonſtrated by ſome, that the Power will be equivalent to the Reſiſtance of the Weight, when it bears ſuch Proportion to it, as the Breadth of the Back of the Wedge

does to the Sum of its Sides; or, which is the ſame Thing, as half that Breadth to one of its Sides. 2. Others make it ſomewhat larger, and demonſtrate, that it ought to be, as half the Breadth of the Back to the perpendicular Height of the Wedge. 3. Some are of Opinion, that there will not be an Æquilibrium in this Machine, unleſs the Power is to the Weight, as the whole Breadth of the Back to the perpendicular Height; *viz.* Wallis, Keil, &c. 4. Gravesande in his *Elements* (L. I. Chap. 13.) gives us the ſame Proportion with the laſt; and in his *Scholium de ligno findendo*, tells us, that when the Parts of the Wood are ſeparated no farther than the Wedge is driven in, the Æquilibrium will be, when the Power is to the Reſiſtance, as half the Breadth of the Back of the Wedge to one of its Sides.

Those, who lay down the firſt Proportion for determining the Power of this Machine, ſuppoſe the Parts, which are ſeparated from each other thereby, to recede from their firſt Situation in Directions perpendicular to the Sides of the Wedge. Thus let ACB (*Fig.* 34.) repreſent a Wedge; P, P, two Bodies to be ſeparated by it, the one to be moved towards I, the other towards F, in the Directions CI, and CF perpendicular to AC and CB; then 'tis evident, that when the Wedge is driven in to the Situation MNO, the two Bodies will be moved to Q and Q; that is, one will have

have paſſed through the Space CK, the other through CL, but theſe Spaces being equal, their Velocities are the ſame as if they had both paſſed over one of them, *v. g.* CL, or which is equal to it DG (drawn perpendicular to CB); therefore the Power, which we ſuppoſe applied at D, moves through DC, while the Obſtacle moves through DG, conſequently (§. 7.) when the Power is to the Weight as DG to DC, that is, as DB to CB*, or half the Back of the Wedge to one of its Sides, they will be in Æquilibrio. This Proportion therefore, when the Parts of the Weight are ſuppoſed to be moved by the Wedge in the Directions CI and CF, is true.

2. The ſecond Proportion is alſo true, ſuppoſing the Bodies P, P, to recede from each other in the Directions CN and CM, parallel to AB the Back of the Wedge; for, when the Wedge is driven in between them, to the Situation MNO, the Bodies will have moved through a Space as CN, or which is equal to it DB, half the Back of the Wedge, and the Power through a Space equal to its Height as before; conſequently (§. 7.) in this Caſe, the Æquilibrium will be, when the Power is

* For (8 *Elem.* 6.) the Triangles *DCG* and *DCB* are ſimilar, and conſequently *DG* : *DC* :: *DB* : *CB*.

to the Weight, as half the Back of the Wedge to its Height*.

3. THOSE, who imagine there will not be an Æquilibrium, unleſs the Power be to the

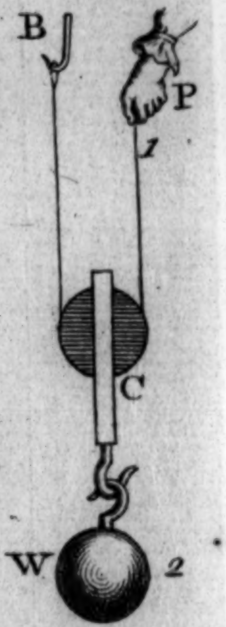
* The ſame may be otherwiſe demonſtrated from Section 14. Chapter 9. thus. Let there be a Body as *L* (*Fig.* 35.) drawn againſt the Wedge *ABC* by the Weight W, in the Direction *LF*, parallel to the Back of the Wedge *AB*; but prevented from ſliding down towards *C*, by a Plane (whoſe upper Surface we may ſuppoſe repreſented by *EF*) lying under it. I ſay, the Power will be to the Weight, when they are in Æquilibrio, as *DA* to *DC*.

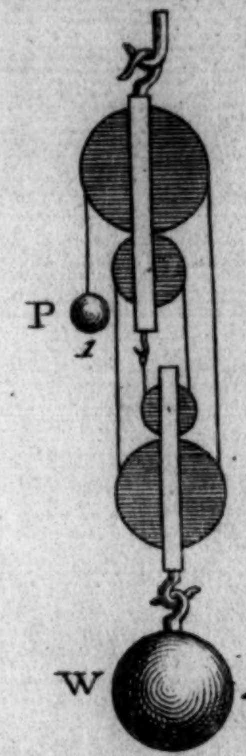
Dem. The Body *L* is here acted upon in three Directions, *viz.* by the Force of the Weight *W* in the Direction *LF*, by the two Planes *CA* and *EF*, in the Directions *LG* and *LI*, perpendicular to their Surfaces; let *GE* be drawn parallel to *LI*, then will the Triangle *LGE* have all its Sides reſpectively parallel to thoſe Directions; conſequently (Chap. 9. §. 14) if we ſuppoſe *LE* to expreſs the Force of the Weight, *GE* will expreſs that of the ſuppoſed Plane, or which is the equal to it, becauſe they act in contrary Directions to that of the Power: But *GE* is to *EL*, as *DA* to *DC* (for the Triangles *EGL* and *DAC* are ſimilar, the Sides of one being *ex Conſtruct.* reſpectively perpendicular to thoſe in the other; v. g. *LG* to *CA*, *EL* to *DC*, and *GE* to *DA*); conſequently the Power is to the Weight, when they balance each other, as half the Breadth of the Back of the Wedge to its Height. *Q. E. D.*

Corol. Suppoſe the Body *L* had been drawn againſt the Wedge in the Direction *GL* perpendicular to its Surface, and were to be moved by it in the contrary Direction towards *G*, as in the firſt Caſe; then if *GL* expreſſes the Force with which it is drawn towards the Wedge, *GE* will be that with which it reſiſts the Power; but *GE* is to *GL* as *DA* to *AC*, the Triangles *EGL* and *DAC* being ſimilar; conſequently in this Caſe, the Power will be to the Weight, as half the Breadth of the Back of the Wedge to one of its Sides; as was before demonſtrated.

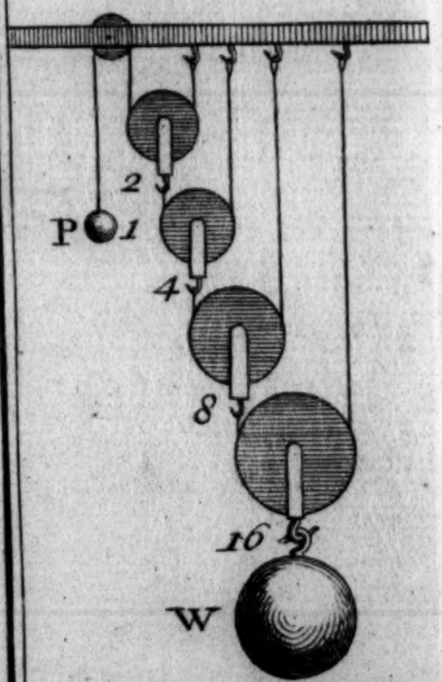
Weight,

F. 30. P. 65. F. 31.

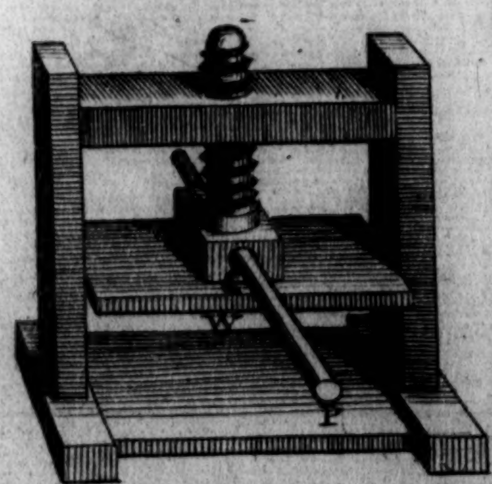




F. 32.



F. 33. P. 66.



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Weight, as the whole Breadth of the Back of the Wedge to its Height, ſuppoſe, as in the laſt Caſe, that the Bodies to be ſeparated recede from each other in Directions parallel to the Back of the Wedge; and endeavour to ſupport their Opinion by the following Argument: *viz.* that, when the Wedge is driven in to the Situation MNO (*Fig.* 34.) as before, each Part of the Weight having moved through a Space equal to half the Back of the Wedge, the whole Weight has therefore moved through twice ſo much, or a Space equal to the whole Back: as much as to ſay, the Whole has moved farther than its Parts; which is abſurd.

4. This is Gravesande's Miſtake in his *Elements*, the ſame he has alſo made in his *Scholium de ligno findendo*, and thereby determined the Power in both Places to be twice as big, as it ought to be. If he had proceeded in the following Manner, his Argument would have been eaſier, as well as the Concluſion juſter. Suppoſe the Wedge ABC driven into the Wood QLQ (as repreſented *Fig.* 36.) which is ſplit no farther than the Point of the Wedge (or however no farther than is juſt ſufficient to give it Room to move) which Caſe Gravesande ſuppoſes in his *Scholium*, I ſay, that in this Situation of the Wedge, the Power is to the Weight, as one fourth Part of the Back of the Wedge to one of its Sides. For it is evident, that when the upper Ends

Ends of the Wood, which preſs againſt the Wedge in the Points G, H, are put into Motion by the Wedge, they will move in the Directions HI and GF, perpendicular to the Sides of the Wedge, becauſe they turn as it were upon a Joint at L, which we always ſuppoſe contiguous to C: again, ſince only the upper Ends of the Wood are put into Motion, and not the lower ones, which remain at L; 'tis evident, that the Motion of each Piece (ſuppoſing their Thickneſs the ſame from End to End, and their Subſtance uniform) will be but half, what it would have been had the lower ones moved with the ſame Degree of Velocity. Now, were all the Parts of the Wood to have the ſame Degree of Velocity with the upper ones, the Power would be to the Weight, as in the firſt Caſe, *viz.* as DB to BC (*Fig.* 34.); therefore in this Caſe, it is as half DB to BC, or as one fourth Part of the Back of the Wedge to one of its Sides. Which was to be proved.

XX. The Form of the *Inclined Plane* being no other than that of half a Wedge, as is manifeſt from the Repreſentation of it (*Fig.* 37.) it follows that what has been demonſtrated of the one, may be applied to the other, and the Properties of both will be found the ſame. For Inſtance, if the Weight W is to be raiſed up the Plane CB, by the Power P, in a Direction parallel to the Plane, inſtead of that, we may ſuppoſe the Weight prevented from

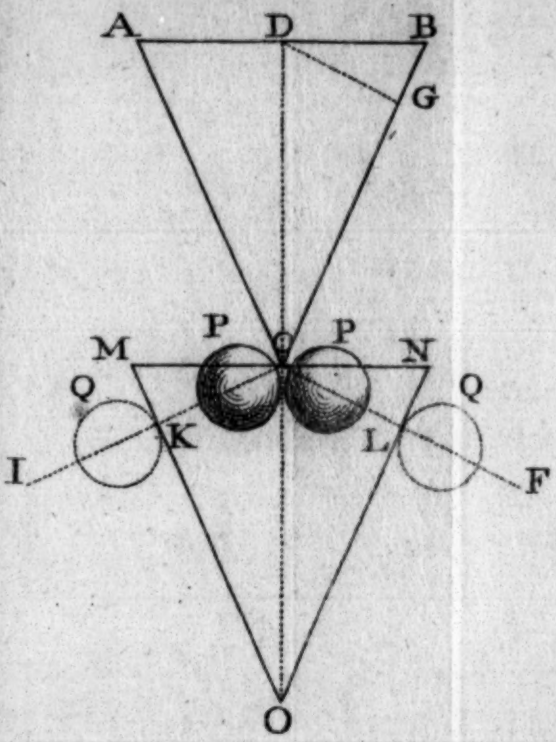
running off the Plane by the String WH, and the inclined Plane driven under it like a Wedge in the Direction DC: then will the Weight rise towards G in a Direction perpendicular to CB, for we must always suppose the String WB parallel to the Plane (as it would have been, if the Weight had been drawn up by it;) then will the Action of the Plane upon the Weight be similar to that of the Wedge in the first Case and consequently the Power will bear such Proportion to the Weight, as DB to BC; that is, as the Height of the Plane to its Length. Again, suppose the Weight was to have been drawn up the Plane by a String in the Direction WF parallel to CD the Base of the inclined Plane; then if the Plane be driven under the Weight as before, it must rise in a Direction perpendicular to CD, that is, parallel to DB: then the Case will be analagous to the second of the Wedge; consequently, the Power will be to the Resistance of the Weight, when there is an Æquilibrium, in the Proportion of DB to DC, as there demonstrated.

XXI These are the Powers or Machines, which, under different Forms, constitute all other how complicated soever; and as the Æquilibrium, in any one of these is, when the Power and Weight are inversely as their Velocities; so in a Machine however compounded, the Power and the Weight will exactly

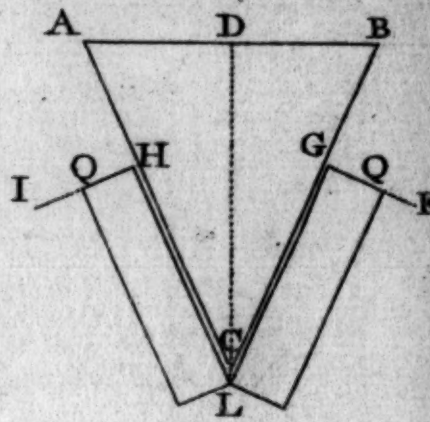
balance each other, when they are in this Proportion; for by §. 7. their Moments will then be equal, and the Machine, if at reſt, will continue in that State; and, if put into Motion by an external Force will gradually loſe it, when that Force ceaſes to act; on account of the unavoidable Friction of the Machine, and the Reſiſtance of the Air, which it muſt neceſſarily meet with, unleſs its Motion could be performed in a perfect Vacuum. From hence we ſee the Impoſſibility of contriving an Engine, whoſe Motion ſhould be *perpetual*, that is, ſuch as does not owe its Continuance to the Application of ſome external Force; a Problem that has given Birth to an almoſt infinite Number of Schemes and Contrivances. For unleſs ſome Method could be found out of gaining a Force, by the artful Diſpoſition and Combination of the Mechanical Powers, equivalent to that which is continually deſtroyed by Friction, and the Reſiſtance of the Air, the Motion, which was at firſt given to the Machine, muſt at length be neceſſarily loſt. But we ſee, that thoſe Inſtruments are only different Means, whereby one Body communicates its Motion to another, and not deſigned to produce a Force which had no Exiſtence before. 'Tis for want of a due Conſideration of this, that ſo many Mechanical Deſigns have proved abortive, ſo many Engines unequal to the Performance for which they

P.t I. P. 74.

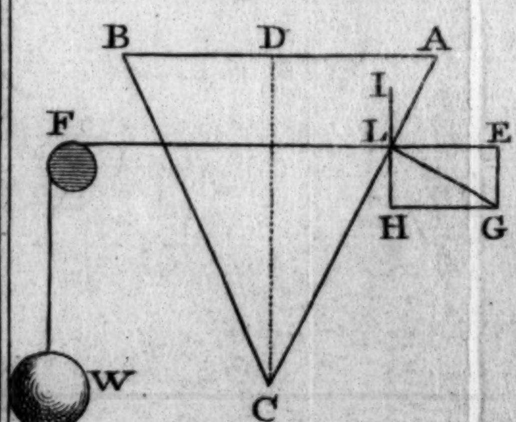
F. 34. P. 68.



F. 36. P. 71.



F. 35. P. 70.



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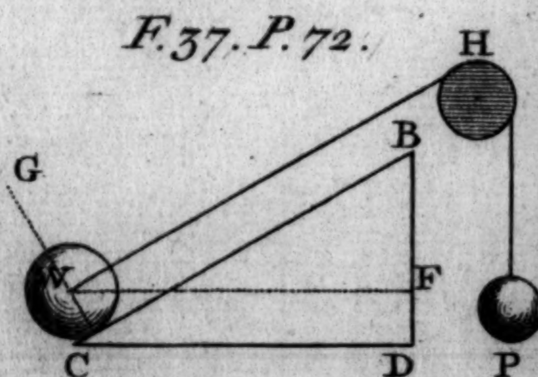

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they were designed, and so many Impossibilities attempted.

"If it were possible, says Bp. Wilkins, " to contrive such an Invention, whereby " any conceivable Weight may be moved by " any conceivable Power, both with equal " Velocity (as it is in those Things which are " immediately stirred by the Hand, without " the Help of any other Instrument) the " Works of Nature would be then too much " subjected to the Power of Art; and Men " might be thereby encouraged (with the " Builders of *Babel*, or the Rebel Giants) to " such bold Designs, as would not become " a created Being. And therefore the Wisdom " of Providence has so confined these Human " Arts, that what an Invention hath, in the " *Strength* of its Motion, is abated in the *Slowness* of it; and what it has, in the extraordinary *Quickness* of its Motion, must be allowed for in the great *Strength* requisite in " the Power which is to move it *.

* *Wilkins*'s Mathem. Magick. p. 104.

APPENDIX to Part I.

CHAP. I.

Of the Vibration *of a* Pendulum *in a* Cycloid.

PROPOSITION I.

IF a Pendulum be made to vibrate in a Cycloid, all its Vibrations however unequal, will be *isocronous*; that is, they will be performed in equal Times (*a*).

(*a*) In order to demonstrate this Proposition, it will be proper to lay down the following Lemma's.

LEMMA I.

If a Body descends from A along the Line AX, (Appendix Plate, Fig. 1.) by virtue of a Force which decreases in Proportion as the Distance of the Body from X, decreases; that is, if when the Body comes to M, N, O, *&c.* the Action of that Force upon the Body, be as the Distances XM, XN, XO, *&c.* respectively: And if the last acquired Velocity of the Body; that is, its Velocity when i tcomes to X, be expressed, or set off, by the Perpendicular XB equal in Length to the Line AX, and its Velocities at M, N, O, *&c.* be set off there by the Lines MD, NP, OQ, *&c.* in Length proportionable to each other and to the Line XB, as the Velocities of the falling Body at M, N, O, *&c.* are to each other and to its last Velocity at X: And if through the Extremities of these Lines, the Curve ADB be drawn; I say, that Curve will be a Portion of a Circle: And the Time in which the Body will descend through the whole Space or Line AX [or any Part of it, as MO] will be such Time, as would be requisite for it to describe the whole Arch AB [or

any Part as DQ, correſponding to MO] in, with its laſt acquired Velocity at X.

Demonſtration of the Lemma. Parallel and contiguous to the Line MD, draw NP, in which Caſe the Line MN, becomes a Point, and the Arch DP a Tangent to the Curve: Produce PD till it meets XA produced, in T; draw the Line XD; and let fall the Perpendicular DL. Then the Lines DL and TM being parallel, the Angles PDL and DTM are equal, as being alternate (by 27 Elem. 1.); and the Angles at L and M, as being right ones; the Triangles therefore PDL and DTM are ſimilar, which for the Sake of referring to it afterwards, let us make the

firſt Step of the following Proceſs	1	The Triangles PDL and DTM are ſimilar
From the firſt Step we have this Proportion (5 Elem. 6.)	2	PL : LD : : DM : MT.
By the Figure	3	LD = MN
But MD being the Velocity when the deſcending Body comes to M, the Point MN is deſcribed with that Velocity; for there is no Acceleration during the Paſſage of a Body over a Point; conſequently MN is proportionable to MD: that is,	4	MN is as MD.
Comparing the ſecond, third and fourth Steps	5	PL : MD : : DM : MT
But MD and NP being the Velocities of the deſcending Body at M and N, LP the Difference of thoſe Lines, expreſſing the Increaſe of Velocity in the Body, will be proportionable to the moving Force at the Point MN; that is, by the Suppoſition, to the Diſtance XM; therefore	6	PL is as XM
Comparing the fifth and ſixth	7	XM : MD : : DM : MT
Conſequently (5. Elem. 6.)	8	The Triangles XMD and DMT are ſimilar.

And therefore, ſince their Angles at M are right ones, the Triangle TDX is (by the Converſe of Prop. 8. Elem. 6.) right angled at D. Conſequently ſince the ſame is true of any other Point of the Curve, as well as D, the Arch ADB is a Portion of a Circle (16 Elem. 3.). Which is the firſt Part.

Secondly,

Secondly, comparing the firſt and eighthSteps, the Triangles PDL and XMD are ſimilar; therefore	9	LD : DM : : DP : DX
ADB being a Portion of a Circle, as already proved	10	DX = XB
Comparing the 3d, 9th and 10th Steps	11	MN : DM : : DP : XB.

Since then the Point MN bears the ſame Proportion to MD, or the Velocity it is deſcribed with by the falling Body, that the Point DP does to the laſt acquired Velocity XB, it follows that the former, MN, is deſcribed in the ſame Time with the Velocity the Body has when there, that the latter, DP, might be with the laſt acquired Velocity XB. And ſince the ſame is true of every other Part of the Arch ADB, it is obvious that the Time in which the Body will deſcend through any other Part of the Space AX, [or the whole of it,] will be ſuch as would be required for it to deſcribe any correſponding Part of the Arch ADB, [or the whole of it,] with the laſt acquired Velocity XB. Which was the other Part.

Coroll. Hence it follows, that if a Body deſcends along the Line AX, by Virtue of Forces acting upon it at A, M, N, O, *&c.* proportionably to the Length of the Lines XA, XM, XN, XO, *&c.* and if on X as a Center, and with the Radius XA a Portion of a Circle, as ADB, be deſcribed; and if the Radius or whole Sine XB, be put to repreſent the Velocity of the Body when it comes to X, the other Sines MD, NP, OQ, *&c.* will repreſent the reſpective Velocities of the Body at the ſeveral Points M, N, O, *&c.* And converſely, if one of the Sines, as MD, be put to expreſs its Velocity at M, the other Sines NP, OQ, and the Radius or whole Sine XB, will expreſs the Velocity of the Body at thoſe other Points N, O and X.

LEMMA II.

If a Body moves along the Line AX, (Fig. 2.) and be urged all the Way by Forces proportionable to its Diſtance from the Point X; whatever Point of that Line it ſets out from, it will come to the Point X in the ſame Time. Which Time will bear ſuch Proportion to the Time it would move over the whole Line AX in, with the Velocity it ſhall acquire by falling through the whole Line AX, as the Semicircumference of a Circle does to its Diameter.

Dem. Let two Bodies A and P ſet out from the Points A and P at the ſame Time; and let them be urged by Forces proportionable to their Diſtances from the Point X: I ſay, thoſe Bodies will come to X at the ſame Inſtance of Time; that is, they will overtake one another at that Point. On X as a Center, and with the Radius's XA and XP deſcribe the two Quadrants AB and PQ; and draw the Line SX, and the Sines RS and MN; and let the whole Sine or Radius XB expreſs the Velocity the Body A will acquire by falling to X: Then by Corollary of Lemma 1. will the Sine RS, if taken as near as poſſible to A, expreſs the firſt Velocity of the Body A. But the Force, which urges the Body A is ſuppoſed to be to that which urges the Body P, as XA to XP (or becauſe the Archs AS and PN are ſimilar) as RS to MN: As therefore RS expreſſes the firſt Velocity of A, MN will expreſs the firſt Velocity of the other Body P: And therefore by the ſame Corollary, its Velocity when it comes to X, will be expreſſible by XQ. Farther, the Time the Body A falls to X in, is by Lemma 1. equal to the Time the Arch AB would be deſcribed in with the Velocity XB; and the Time the other Body falls from P to X in, is equal to the Time the Arch PQ would be deſcribed in, with the Velocity XQ. But a Body will be as long in moving over the Arch PQ with the Velocity XQ, as over the Arch AB with the Velocity XB, the Lines XQ and XB having the ſame Proportion to each other, that the Archs have. Therefore the Time the Body A falls to X in, is equal to the Time the other Body P would fall to that Place in. Which was the firſt Part.

Again by Lemma 1.	1	The Time a Body would fall from A to X in, is equal to the Time it would move over the Arch AB in, with its laſt acquired Velocity at X.
Axiom, or ſelf evident Propoſition	2	The Time a Body would move over the Arch AB in with the laſt acquired Velocity at X, is to the Time it would move over AX in with the ſame Velocity, as AB is to AX.
Comparing the firſt and ſecond	3	The Time a Body would fall from A to X in, is to the Time it would move over AX in with the laſt acquired Velocity, as AB is to AX.

Axiom

Axiom	4	AB is to AX as twice AB is to twice AX.
By the Figure	5	Twice AB is to twice AX as the Semicircumference of a Circle is to its Diameter.
Comparing the 3d, 4th and fifth Steps.	6	The Time a Body would fall from A to X in, is to the Time it would move over AX in with its last acquired Velocity, as the Semicircumference of a Circle is to its Diameter. Which was the second Part.

LEMMA III.

If from the lowermost Point of a Circle, as X (Fig. 3.) be drawn the Chords XQ and XO, the Power of Gravity whereby it shall cause a Body to descend along the former, will be to the Power whereby it shall cause it to descend along the latter, as the Length of the former is to the Length of the latter.

Dem. Draw the Diameter XD, the Perpendiculars QR and OS; and join the Points QD and OD. Then (by 31 Elem. 3.) the Triangle XQD is right-angled at Q; and therefore (by 8. Elem. 6.)

Elem. 6.)	1	XR : XQ : : XQ : XD.
And for like Reasons	2	XS : XO : : XO : XD
But by Part I. Chap 6. § 2.	3	The Effect or Power of Gravity upon a Body descending along the Chord QX, is to that which it exerts upon another falling freely; that is, to its whole Power, as XR to XQ.
And also	4	The Power of Gravity upon a Body descending along the Chord OX, is to its whole Power, as XS to XO.
Comparing the 1st and 3d	5	The Power of Gravity upon a Body descending along the Chord QX is to its whole Power, as XQ to XD.
Comparing the 2d and 4th	6	The Power of Gravity upon a Body descending along the Chord OX is to its whole Power, as XO to XD.

Comparing

Comparing the 5th and 6th Steps	7	The Power of Gravity upon a Body descending along the Chord QX, is to the Power of Gravity upon a Body descending along the Chord OX, as XQ to XO. Q. E. D.

The Description of a Cycloid, with the Definitions relating thereto. If a Circle as FCH (Fig. 4.) be rolled along the Line AB, till it has turned once round; the Point C in its Circumference, which at first touched the Line at A, will describe the Curve Line ACXB, which Curve is called a *Cycloid.* The right Line AB is its *Base:* The middle Point X is its *Vertex:* And a Perpendicular, as XD, let fall from thence to the Base, is its *Axis:* And the Circle FCH, or any other as XGD, equal thereto, is called the ***Generating Circle.***

LEMMA IV.

If on XD, the Axis of the Cycloid, as a Diameter, the generating Circle XGD be described; and if from a Point in the Cycloid, as C, the Line CIK be drawn Parallel to the Base, the Portion of it CG, will be equal to the Arch GX.

Dem. Draw the Diameter HF, then the Circles FCH and

DGX being equal	1	KG = CI
Adding GI to each of them	2	KI = CG
By the Figure	3	KI = DF
Comparing the two last	4	CG = DF
By the Description of the Cycloid	5	The Arch CF = AF
By the Figure	6	The Arch CF = DG
Comparing the 5th and 6th	7	AF = DG
By the Description of the Cycloid	8	AFD = DGX
Comparing the 7th and 8th with the Figure	9	FD = GX
Comparing the 4th and the 9th	10	CG = GX. Q. E. D.

LEMMA V.

The same Things being supposed as in the foregoing Lemma, a Tangent to the Cycloid at the Point C, is parallel to GX a Chord of the Circle DGX.

Dem. It appears from the Description of the Cycloid, that since the Angle FCH is a right one, (as it is by 31 Elem. 3.) the Chord CH is a Tangent to the Curve at the Point C, but CH

CH is parallel to GX; a Tangent therefore at the Point C, is parallel to GX, the Chord of the Circle DGX. Q. E. D.

LEMMA VI.

Things remaining as before, if from a Point of the Cycloid, as L, the Line LMK be drawn parallel to the Base AB, the Arch XL of the Cycloid, will be double of XM the Chord of the Circle correſponding thereto.

Dem. Draw the Line SQ parallel and contiguous to LK, croſſing the Circle in R, and the Chord XM produced, in P, then will LS, MR and MP become Points, the firſt having the Property of a Tangent to the Cycloid at LS, the ſecond that of a Tangent to the Circle at MR, and the third, the Properties of a Production of the Chord XM. Join the Points X and R, and on MP let fall the Perpendicular RO: Produce alſo the Point RM, till it meets XN, a Tangent to the Circle at X. Then will the Lines XN and QS, being each perpendicular to the Diameter DX, be parallel; and the Triangles MNX and MPR will be ſimilar; as having their Angles at M vertical, and at P and X alternate. But the Tangents NX and NM are equal (by 36. Elem. 3.) the correſponding Lines therefore PR and RM in the other Triangle, are ſo too: This laſt Triangle is therefore an Iſoſceles one; and therefore RO being perpendicular to its Baſe MP, MP is equal to twice MO. The Tangent LS is parallel to MP, (as being by Lemma 5. parallel to MX) and therefore equal to it, the Lines LK and SQ being parallel: It is therefore equal alſo to twice MO. But LS is the Difference between the cycloidal Archs XL and XS; and MO is the Difference between the Chords XM and XR, for ſince XO and XR are cloſe together, RO which is perpendicular to one of them, may be conſidered as perpendicular to both: The Difference therefore between any two Archs of the Cycloid is twice that which is between two correſponding Chords of the Circle; and conſequently any Arch, as XL, is double of the correſponding Chord XM. Q. E. D.

Coroll. Since when the Arch XL becomes XB, the correſponding Chord XM becomes XD the Diameter of the Circle DMX; its obvious, that the Semicycloid BX, or AX, is equal to twice DX the Diameter of the generating Circle DMX.

LEMMA VII.

If a Body deſcends in a Cycloid, the Force of Gravity (ſo far as it acts upon it in cauſing it to deſcend along the Cycloid) will

be

PROPOSITION II.

The Time in which a Pendulum vibrating in a Cycloid, performs a Vibration, is to the

be proportionable to the Diſtance of the Body from the loweſt Point of the Cycloid.

Dem. Let the Cycloid be AXB (Fig. 5.) whoſe Baſe is AB, and its Axis DX, on which laſt as a Diameter, deſcribe the generating Circle DQX: Draw the Chords OX and QX; through the Points O and Q, and parallel to the Axis AB, draw the Lines LS and MR; draw alſo the Tangents LV and MY. Then becauſe by Lemma 5. the Tangent LV is parallel to OX, and the Tangent MY parallel to QX, its obvious that Gravity exerts the ſame Power or Force upon a Body deſcending in the Cycloid at L (becauſe it then deſcends in the Tangent LV) as it would do upon the ſame Body deſcending along the Chord OX: And for the like Reaſon, it exerts the ſame Force upon it when it comes to M, that it would do if it were deſcending along QX: But (by Lemma 3.) the Power or Force of Gravity upon Bodies deſcending along the Chords OX and QX, are as the Lengths of thoſe Chords; that is, by Lemma 6. (halves being proportionable to their wholes) as the Length of the Cycloidal Archs LX and MX. The Force therefore of Gravity upon a Body deſcending in the Cycloid at the Point L [or any other] is to its Force upon the ſame when at M [or any other Point] as the Space or Diſtance it has to move over in the former Caſe, before it gets to the loweſt Point X, to that it has to run over in the latter, before it arrives at the ſame Point. Q. E. D.

Demonſtration of the Propoſition in the Text to which this Note refers.

By Lemma 7. The Force of Gravity ſo far as it cauſes a Body to deſcend in a Cycloid is proportionable to the Diſtance of that Body from the loweſt Point; imagine then that Body to be a Pendulum vibrating in the Cycloid, then whatever Point it ſets out from, it will by Lemma 2. come to the loweſt Point in the ſame Time: And conſequently ſince the like is true as to its aſcending from that Point, all its Vibrations be they large or ſmall, will be perform'd in the ſame Time. Q. E. D.

Time

Time in which a Body would fall freely thro' half the Length of the Pendulum, as the Circumference of a Circle is to its Diameter (*b*).

PROB.

(*b*) To demonstrate this, the following Lemma's will be of Use.

LEMMA VIII.

If in a right-angled Triangle, as BFG (Fig. 6.) the Perpendicular FI be let fall from the right Angle to the Hypothenuse BG, the Line BI multiplied by BG will be equal to BF^q.

Dem. By 8. Elem. 3. the Triangle BFI and BFG are similar, consequently BI is to BF, as BF is to BG, and therefore $BI \times BG = BF^q$. Q. E. D.

LEMMA IX.

If a Body descends along a curve Line, as AX (Fig. 7.) it will acquire the same Velocity that another, or the same Body, would do, by falling from an equal perpendicular Height in the Line DX.

Dem. Parallel to the horizontal Line AD, draw the Lines BM and FN contiguous to each other; in consequence of which, the Lines MN and BG are capable of being considered as Points; and therefore the Velocity the descending Bodies pass over them with, as uniform; and the curve Line BG, as a straight Line also, and as a Tangent to the Curve AX at the Point BG. Things being thus, let it be supposed that the Bodies begin their Fall at B and M, or, which comes to the same Thing, that they have equal Velocities at those Points: Then the Velocities of the Bodies being uniform and equal to each other, (for there is no Acceleration in a Point) the Lines BG and MN may represent the Relation the Times they are passed over in bear to each other. Parallel to DX draw BF, and let the equal Lines BF and MN represent the Force of Gravity acting perpendicularly at those Points; and let the Force BF be resolved into two others, *viz.* BI and IF, the one parallel, the other perpendicular to the Curve of the Body at B: It is only the former of these, *viz.* BI, that accelerates the Body along the Curve BG; the other, *viz.* IF, neither accelerates it nor retards it, but is wholly spent in pressing the Body close to the Surface BG, if it be a Surface; or in stretching the String which keeps the Body in the Course ABX, if it be a String. Now the Velocity a Body acquires by moving over any Space, is proportionable to the Force that acts

upon it, multiplied by the Time that Force acts. Since then BI represents the Force in one Case, and MN the Time in the other, it follows that the Velocity generated in one Case, is as BI × BG; and in the other, as MN × MN; or since BF and MN are equal, as the Quantities BI × BG and BF × BF, (or BF^2) which Quantities by Lemma 8. are equal to each other. The Velocity therefore the one Body acquires by descending along BG, is equal to that which the other acquires by falling through MN: But the Lines BM and GN being parallel, it is obvious there is the same Number of BG's in the Curve AX, as of MN's in the perpendicular DX; the Velocity therefore which a Body would acquire by falling through one, is equal to that which it would acquire in falling through the other. Q. E. D.

Demonstration of the Proposition. Let AXB (Fig. 5.) be the Cycloid the Pendulum vibrates in. Then by Lemma 2. compared with Lemma 7, we have

	1	The Time a Body would descend from A to X in, is to the Time it would move over the same Space in with its last acquired Velocity, as the Semicircumference of a Circle is to its Diameter.
By the Corol. of Lemma 6.	2	AX is equal to twice DX.
By Lemma 9.	3	The Velocity a Body acquires by falling from A to X, is equal to the Velocity it would acquire by falling from D to X.
From the three last compared	4	The Time a Body would descend from A to X in, is to the Time it would move over twice DX in, with the Velocity acquired by a Fall from D to X, as the Semicircumference of a Circle is to its Diameter.
By Part I. Chap. 5. § 7.	5	The Time a Body would move over twice DX in, with the Velocity acquired by falling from D to X, is equal to the Time it would fall from D to X in.
Comparing the 4th and 5th	6	The Time a Body would descend from A to X in, is to the Time it would fall from D to X in, as the Semicircumference of a Circle is to its Diameter.

From

PROBLEM.

To make a Pendulum *vibrate in a given* Cycloid.

Solut. Let AXB (Fig. 5.) be the given Cycloid; its Base AB, its Axis DX, and its generating Circle DQX, as before: Produce XD to C, till DC be equal to DX: Through C draw the Line EF parallel to AB, and take CE and CF, each equal to AD or DB; and on the Line CE as a Base, and with the generating Circle AGE equal to DQX, describe the Semicycloid CTA, whose Vertex will therefore touch the Base of the given Cycloid in A.

Prom the Figure	7	The Time of Descent from A to X is half a Vibration.
From the Solution of the following Problem it will appear, that	8	DX is half the Length of a Pendulum, which in vibrating shall describe the Cycloid AXB.
Comparing the three last Steps	9	The Time of half a Vibration is to the Time in which a Body would fall freely through half the Length of the Pendulum, as the Semicircumference of a Circle is to its Diameter.
Doubling the Antecedents of the last Step	10	The Time of an whole Vibration is to the Time in which a Body would fall freely through half the Length of the Pendulum, as the Circumference of a Circle is to its Diameter. Q. E. D.

 And

And on the Line CF also as a Base, describe an equal Semicycloid CB. On the Point C, hang the Pendulum CTP equal in Length to the Line CX: And let the upper Part of the String of it, (as CT, in its present Situation in the Figure) as it vibrates this way and that, apply itself to the cycloidal *Cheeks* CA and CB: Then will the Ball of it P oscillate in the given Cycloid AXB. Q. E. F. *(c)*.

CHAP.

(c) Draw TG and PH, each parallel to the Base AB; and join the Points AG and DH. Then by the Corollary of Lemma 6.

	1	AC = 2 AE
By the Figure (DC being equal to DX)	2	2 AE = CX
Comparing the 1st and 2d Steps	3	AC = CX
By Construction	4	CTP = CX
Comparing the 3d and 4th	5	AC = CTP
From the 5th Step compared with the Figure	6	AT = TP
(The String touching the Cycloid at T) by Lemma 5.	7	GA is parallel to TK
By Construction	8	GT is parallel to AK
From the two last Steps compared, GATK is a Parallelogram, consequently	9	GA = TK, and GT = AK
By Lemma 6.	10	GA = $\frac{1}{2}$ TA
Comparing the two last Steps	11	TK = $\frac{1}{2}$ TA
Comparing the 6th and 11th	12	TK = $\frac{1}{2}$ TP
From the 12th Step compared with the Figure	13	TK = KP
Comparing the last Step with the Figure	14	The parallel Lines GT and PH are equally distant from AD
From the last compared with the Figure	15	The Arch GA = the Arch DH
Comparing the last with the Figure	16	The Chords GA and DH are parallel, and GE = HX.

From

CHAP. II.

Of the Center *of* Oscillation *and* Percussion.

THE Center of Oscillation is that Point in a Pendulum, in which, if the Weight of the several Parts thereof were collected, each Vibration would be performed in the same Time, as when those Weights are separate.

The Point or Center, of Suspension is the Point on which the Pendulum hangs.

A general Rule *for finding the* Center *of* Oscillation.

If several Bodies be fixed to an inflexible Rod suspended upon a Point, and each Body

From the 7th and 16th Steps compared with the Figure	17	KP is parallel to DH.
And therefore (KD being by Construction parallel also to PH) KDHP is a Parallelogram, consequently	18	KD = PH
By Lemma 4.	19	GT = the Arch AG
Comparing the 9th and 19th	20	AK = the Arch AG
By the Description of the Semicycloid CTA	21	AKD = AGE
From the two last compared with the Figure	22	KD = GE
Comparing the 18th and 22d	23	PH = GE
Comparing the 16th and 23d	24	PH = HX.

But by Lemma 4. if PH be equal to HX, P is a Point in the Cycloid AXB; the Ball of the Pendulum CTP therefore being at that Point, is in the given Cycloid. The Problem therefore was rightly solved. Q. E. D.

be

be multiplied by the Square of its Diſtance from the Point of Suſpenſion, and then each Body be multiplied by its Diſtance from the ſame Point; and all the former Products when added together, be divided by all the latter Products added together, the Quotient which ſhall ariſe from thence, will be the Diſtance of the Center of Oſcillation of thoſe Bodies from the ſaid Point.

Thus, if CF Fig. 8. be a Rod on which are fixed the Bodies A, B, D, *&c.* at the ſeveral Points A, B, D, *&c.* and if the Body A be multiplied by the Square of the Diſtance CA, and B be multiplied by the Square of the Diſtance CB, and ſo on for the reſt: And then if the Body A be multiplied by the Diſtance CA, and B be multiplied by the Diſtance CB, and ſo on for the reſt; and if the Sum of the Products ariſing in the former Caſe, be divided by the Sum of thoſe which ariſe in the latter, the Quotient will give CQ, the Diſtance of the Center of Oſcillation of the Bodies A, B, D, *&c.* from the Point C *(d)*.

(d) Dem. That the Proceſs may be leſs complicated, let us ſuppoſe but two Bodies, as A and F, fixed to the Rod CF; and let AI and FL be the Archs which the Bodies A and F deſcribe when the Pendulum vibrates, and let the Pendulum be removed into the Situation CL. Contiguous to the Line CL draw CR; then may the Archs IP and LR be conſidered as Tangents at the Points I and L, and thoſe Tangents as inclined Planes, down which the Bodies I and L are to roll: Theſe Tangents being each perpendicular to CL, are equally inclined to the Horizon, the Bodies therefore will endeavour to roll down with equal Velocities; but this they cannot do, becauſe being fixed to the inflexible

flexible Rod, they will deſcribe the unequal Arch IP and LR in the ſame Time. That is, the Body L will oblige the Body I to deſcribe a leſs Arch than it otherwiſe would have done; and the Body I will occaſion the Body L to deſcribe a larger Arch than it would have done. And, the Effects of the Forces by which they act thus upon each other, like thoſe of Action and Reaction, will be equal. It remains to determine theſe Effects.

In order to which, parallel to LI draw MN, and let the equal Spaces LM and IN be thoſe the Bodies would move over in the leaſt Time poſſible, had they been independent of each other. And let the Archs LR and IP be thoſe which the Bodies join'd to the Rod deſcribe in the ſame Time. For the Reaſon juſt mention'd, the former of theſe *viz.* LR, will be larger, and the latter, *viz.* IP, will be leſs than LM or IN; and the Arch which the Center of Oſcillation deſcribes will be equal to LM or IN, becauſe the Center of Oſcillation deſcribes that Arch, which the Bodies would deſcribe in the ſame Time, if they were both together, and neither of them an hindrance or furtherance to the other. Conſequently the Center of Oſcillation is at Y, where the Lines MN and PR croſs.

Now the Motion which the Body I loſes by being retarded, is its Motion over the Arch PN; and the Motion the other Body gains by being accelerated, is its Motion over MR: The Force or Moment of the firſt of theſe Motions, is the Product of the Body I multiplied by the Space PN; and the Force or Moment of the laſt is the Product of the Body L multiplied by the Space MR. Theſe are the Forces, Moments or Actions, which retard the one Body, and promote the Motion of the other. But obſerve, that theſe Forces or Moments, in as much as they act at different Diſtances from the Center C, about which the Bodies I and L, when the Pendulum ſwings, do revolve; have each their *Mechanical* Advantage; but the one a greater than the other: For inſtance, L has an Advantage which is as LC, its Diſtance from the *Fulchrum* C; and I only the Advantage IC. As then in determining the Effect of a Power applied to a Lever, we multiply it by its Diſtance from the *Fulchrum*; ſo the abovementioned Forces or Moments (*viz.* I multiplied by PN and L multiplied by MR) muſt be multiplied by their reſpective Diſtances from C; and then we have I multiplied by PN multiplied by IC, and L multiplied by MR multiplied by LC for the Effects, which, as things are circumſtantiated, thoſe Forces or Moments have upon the Bodies I and L. But, as obſerved above, thoſe Effects are equal, conſequently we have for the firſt Step

| i | I × PN

The Center of Percussion is that Point in a Pendulum, or in an inflexible Rod moving

	1	$I \times PN \times IC = L \times MR \times LC$
But the Triangles PNY and MRY are similar, consequently	2	$PN : MR :: PY : RY$
Comparing the two last	3	$I \times PY \times IC = L \times RY \times LC$
Or taking the Pendulum in the Situation CPR, in which I coincides with P, and L with R, we have	4	$P \times PY \times PC = R \times RY \times RC$
Or, which is the same thing	5	$A \times AQ \times AC = F \times FQ \times FC.$

That is, in Words, if one of the Bodies were multiplied by its Distance from the Center of Oscillation, and the Product arising from thence were multiplied by the Distance of the same Body from the Center of Suspension, this last Product would be equal to the Product of the other Body multiplied by its Distance from the Center of Oscillation, multiplied by its Distance from the Center of Suspension. And, since the same would be true if there were more Bodies, if each Body be multiplied by its Distance from the Center of Oscillation, and that Product by the Distance of the same Body from the Center of Suspension, all the Products relating to the Bodies on one Side the Center of Oscillation taken together, will be equal to all those which relate the Bodies on the other Side thereof taken together. Let then the Distances of any Number of Bodies, as A, B, D, F, from the Center of Suspension be called a, b, d, f, respectively, and the Distance of the Center of Oscillation Q from the Center of Suspension C, be called x: And suppose the Distances of the Bodies A, B, D, less than the Distance CQ, or x; and that of the Body F greater, as in the Figure: Then will the Distances of A, B and D from the Center of Oscillation be expressible by $x-a$, $x-b$, and $x-d$; and the Distance of F, by $f-x$; multiplying then each Body by its Distance from one Center, and the Product arising therefrom by the Distance of the same Body from the other Center, we shall have $Aax - Aaa + Bbx - Bbb + Ddx - Ddd = Fff - Ffx$, which reduced gives $x = \frac{Aaa + Bbb + Ddd + Fff}{Aa + Bb + Dd + Ff}$. Which latter Equation is the Sense of the Rule above laid down.

round

round a Point, with which, if the Pendulum or Rod ſtrikes againſt an Obſtacle, no Jar or Shock at the Point of Suſpenſion ſhall be occaſioned thereby.

Thus, let CF (Fig. 8.) be an inflexible Rod, having the Bodies A, B, D, &c. fixed in it at the Points A, B, D, &c. and let O be an Obſtacle againſt which, as it vibrates or ſwings round the Point of Suſpenſion C, it may ſtrike againſt: then, if there be no Jar or Shock occaſioned thereby at the Point C, the Point that ſtrikes againſt O, (as the Point Q ſuppoſe) is called the Center of Percuſſion.

PROPOSITION.

The Center of Percuſſion is the ſame with the Center of Oſcillation; and conſequently may be determined by the ſame Rule *(e)*.

PROB.

(*e*) *Dem.* From the Definition of the Center of Percuſſion above laid down, it appears, that the Forces, with which the Bodies A, B and D, which would paſs above O, move; muſt be a Counterbalance to the Force of the Body F, which would paſs below it: and that the Force of F muſt be a Counterbalance to them. But the Forces wherewith thoſe Bodies move, are as their Maſſes multiplied by their Diſtances from C, their Velocities being as theſe Diſtances. Farther, when the Point Q comes to O, and is ſtopt there, the Bodies A, B and D, endeavouring to go on, ſway or bear againſt F, and F againſt them; juſt as if they were fixed to a Lever, as AF, having its Fulchrum at Q. Conſequently the Forces of the former Bodies, ſo far as they act againſt the latter, are as their Diſtances from the Point Q; and the Force of the latter, ſo far as it acts againſt

PROBLEM.

Let it be required to find the Center of Oſcillation, or Percuſſion of an inflexible Rod AB (Fig. 9. as a Bar of Iron, or the like) every where of equal Size, and vibrating in, or revolving round the Point A, as a Center of Suſpenſion. (*f*)

the former, is as its Diſtance alſo from Q: the abovementioned Forces muſt therefore be multiplied by the Diſtances of the Bodies from Q: but the former of them, as obſerved above, balances the latter; and the latter them. So many therefore of the laſt Products as relate to the Bodies above Q taken together, muſt be equal to that which relates to the Body (or Bodies) below it. But the like Products were equal to each other, when the Point Q was looked upon as the Center of Oſcillation (as in the 5th Step of the foregoing Proceſs) conſequently the Center of Percuſſion is the ſame with that of Oſcillation. Q. E. D.

(*f*) *Solut.* Imagine the Rod to be divided into the leaſt poſſible Parts B, C, D, *&c.* each of which call *One*. Theſe Parts we may conſider as ſo many Bodies contiguous to one another; ſo that the Center of Oſcillation or Percuſſion of theſe Bodies will be the Center of Oſcillation or Percuſſion of the whole Rod. To find this, we are by the Rule above laid down in the Text, to multiply each of theſe Bodies by the Square of its Diſtance from A. The firſt of theſe Products then will be B (or One) multiplied by AB ſquared; but one multiplied by AB ſquared, is the ſame with AB ſquared; now AB ſquared is a ſquare Area or Surface, one of whoſe Sides is AB. In like manner the Body C, when multiplied by the Square of its Diſtance from A, is a Square Area, one of whoſe Sides is AC, ſomewhat leſs than the former. Imagine this Area laid upon the former; and the next, which will be leſs ſtill, laid upon that; and ſo on till you come to the leaſt of all. Theſe will make a Pyramid, whoſe Baſe is the firſt Area, and its perpendicular Height will be equal to the Thickneſs of them all together; which Thickneſs will be as the Length of the Line BA. The Value or ſolid Con-

tent

tent of this Pyramid will be AB_q (*viz.* its Base) multiplied by a third Part of AB (its perpendicular Height). In the next Place we are to multiply each of those Bodies by its Distance from A: Now the Body B (or One) multiplied by AB, give a Line, as AB; so the Product of C, multiplied by its Distance AC, give a Line, as AC; these Lines heaped one upon another (as the Areas were before) will make a Triangle, whose Base will be AB, and its perpendicular Height also AB; the Value, or Area of which, will be AB multiplied by $\frac{1}{2}$ AB. In the last Place, by the Rule, we are to divide the Sum of the Products in the first Case, by the Sum of the Products in the latter; that is, the Content of the Pyramid by the Area of the Triangle; that is, $ABq \times \frac{1}{3} AB$, by $AB \times \frac{1}{2} AB$, which gives $\frac{\frac{1}{3} AB}{\frac{1}{2}}$; that is, $\frac{2}{3}$ AB, or two Thirds of AB: so that the Distance of the Center of Oscillation or Percussion, (as E suppose) from A the Center of Suspension, must be equal to two Thirds of AB, the whole Length of the Rod. Q. E. I.

Appendix to part I.

Plate I.

Fig. 1.

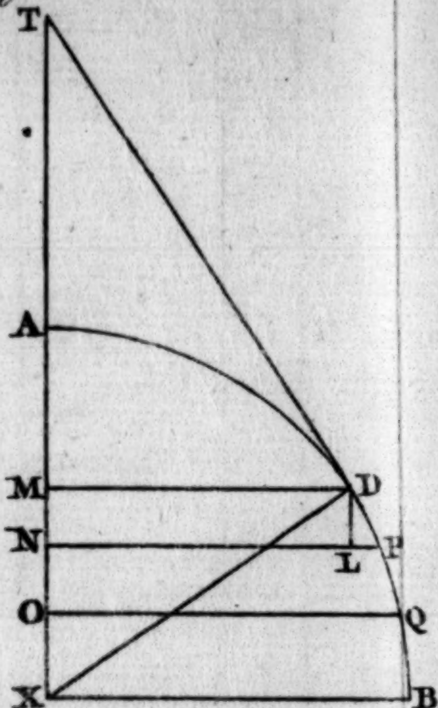

Fig. 2.

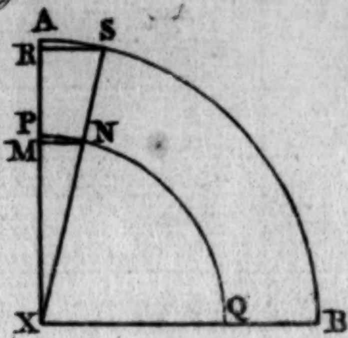

Fig. 3.

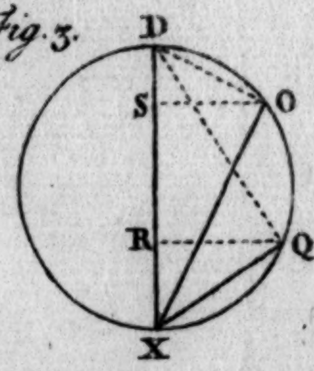

Fig. 4.

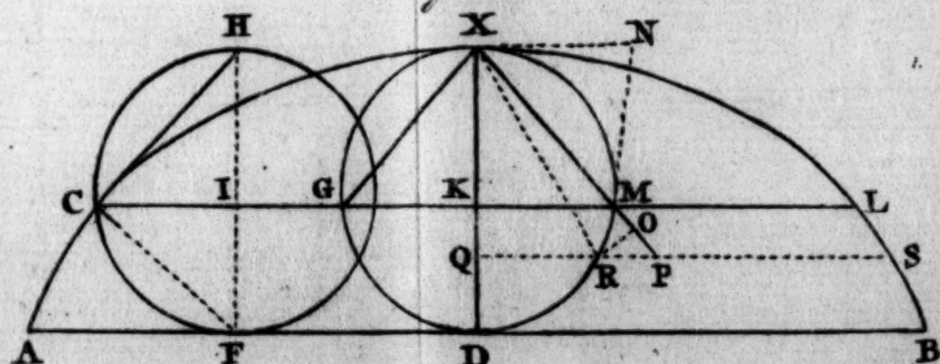

Fig.

Fig. 5.

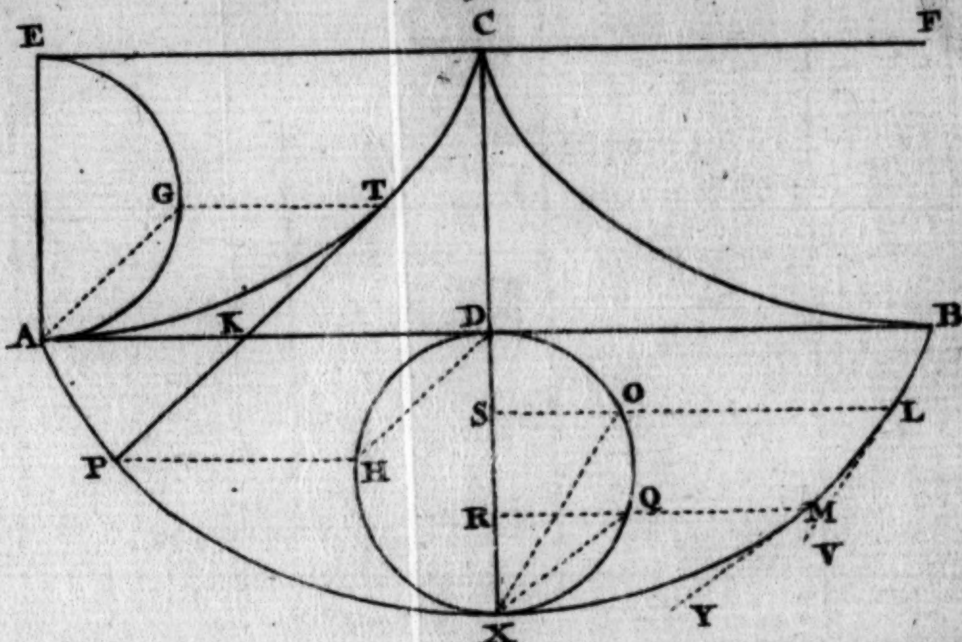

Fig. 6.

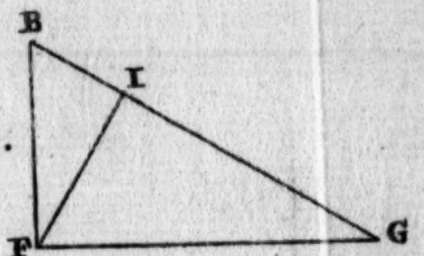

Fig. 8.

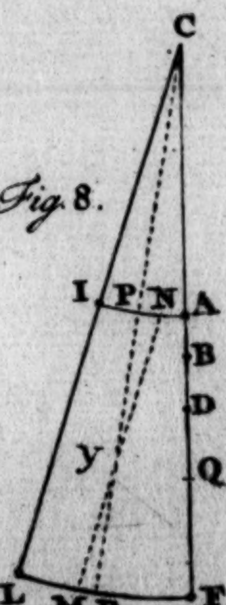

Fig. 9.

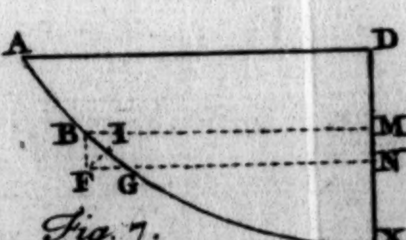

Fig. 7.